Advanced Robotics—Redundant Serial Manipulators

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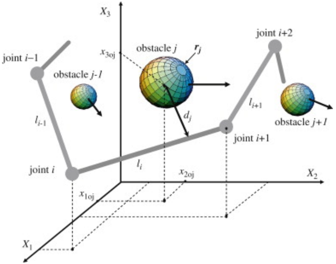
Human and Robot Interaction Laboratory

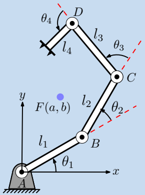


Human and Robot Interaction Laboratory

The Redundant Serial Manipulators

Introduction

- Why redundancy!
- Redundancy in parallel and serial manipulators
- How to solve it



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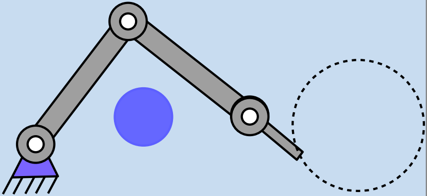

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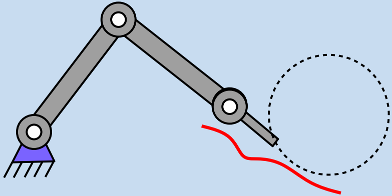

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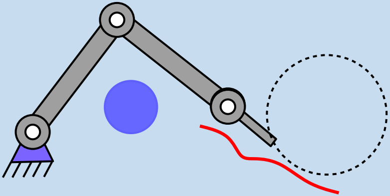

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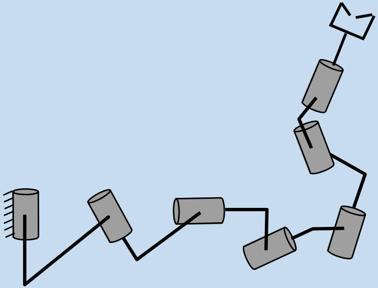

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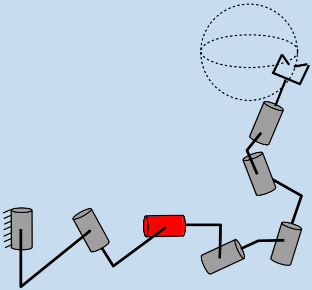

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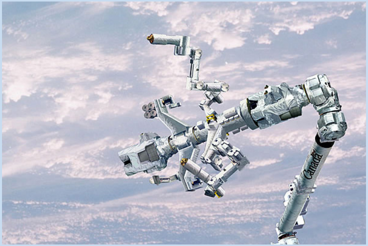

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- Obstacle avoidance
- Singularity avoidance
- Fault tolerance

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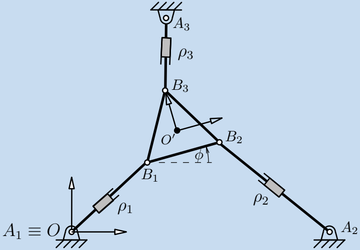

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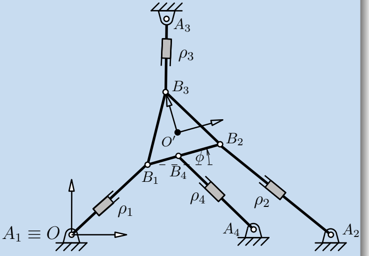

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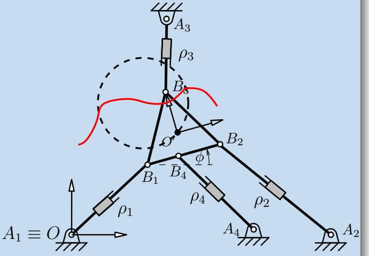

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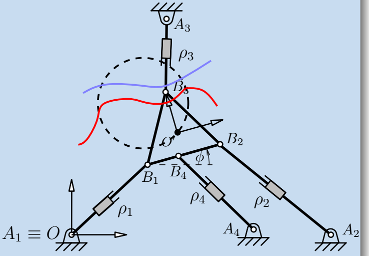

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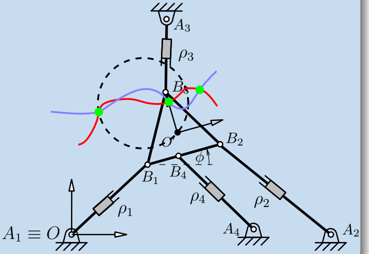

The Redundant Serial Manipulators

Introduction

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- Solve $\mathbf{A}_{m\times n}\mathbf{x}_n = \mathbf{b}_m$
  1. when $m > n$
  2. when $m < n$
- The SVD
- The performance index
- A part of Advanced Mathematic For Eng.

The Redundant Serial Manipulators

Introduction

- Why redundancy!
- Redundancy in parallel and serial manipulators
- How to solve it
- Inverse Kinematic Problem
- Forward Kinematic Problem
- The Singularity
- The workspace

The Redundant Serial Manipulators

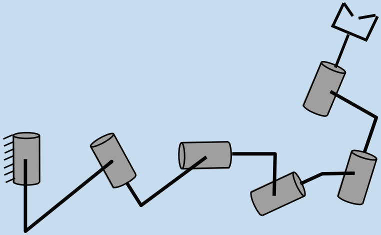
Velocity Equation

- The system of equations to be solved is:

$$\boldsymbol{\theta} = \mathbf{J}\dot{\boldsymbol{\theta}}$$

- Why we have such a relation for the Jacobian:

$$\begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \ldots \mathbf{e}_n \\ \mathbf{e}_1 \times \mathbf{r}_1 & \mathbf{e}_2 \times \mathbf{r}_2 & \ldots \mathbf{e}_n \times \mathbf{r}_n \end{bmatrix}$$



- A review

The Redundant Serial Manipulators

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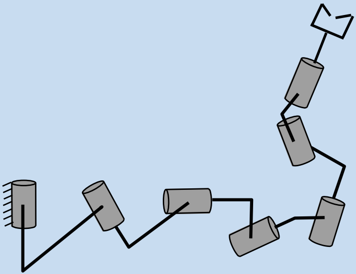

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The Redundant Serial Manipulators

The General Formulation of the Jacobian Matrix

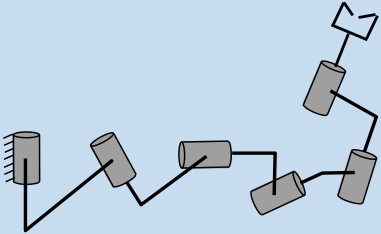
- For the angular velocity, one has:

$$\boldsymbol{\omega}_1 = \dot{\theta}_1 \mathbf{e}_1, \, \boldsymbol{\omega}_2 = \dot{\theta}_1 \mathbf{e}_1 + \dot{\theta}_2 \mathbf{e}_2$$
$$\boldsymbol{\omega} = \dot{\theta}_1 \mathbf{e}_1 + \dot{\theta}_2 \mathbf{e}_2 + \ldots + \dot{\theta}_n \mathbf{e}_n$$

- For $\dot{\mathbf{p}}$, develop the following:

$$\dot{\mathbf{p}} = \dot{\mathbf{a}}_1 + \dot{\mathbf{a}}_2 + \ldots + \dot{\mathbf{a}}_n$$

- Then you should find the Jacobian matrix.



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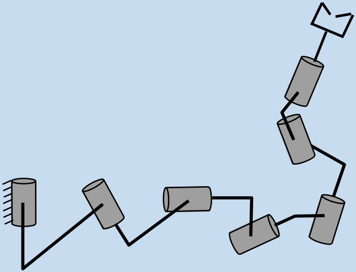

The Redundant Serial Manipulators

Rotation Sequence

- All the rotation matrix are expressed in a fixed frame

$$\mathbf{v}_1 = \mathbf{Q}_1\mathbf{v}$$
$$\mathbf{v}_2 = \mathbf{Q}_2\mathbf{v}_1 = \mathbf{Q}_2\mathbf{Q}_1\mathbf{v}$$
$$\mathbf{v}_3 = \mathbf{Q}_3\mathbf{v}_2 = \mathbf{Q}_3\mathbf{Q}_2\mathbf{Q}_1\mathbf{v}$$

- All the rotation matrix are expressed in their corresponding local frames

$$[\mathbf{A}]_1 = \mathbf{Q}[\mathbf{A}]_2\mathbf{Q}^T$$

The Redundant Serial Manipulators

The Undetermined Linear Case

- For $\mathbf{A}_{m\times n}\mathbf{x}_n = \mathbf{b}_m$
- we have $m < n$
- and the solutions can be expressed as follows:

$$\mathbf{x} = \mathbf{A}^{\dagger}\mathbf{b}$$

- where $\mathbf{A}^{\dagger}$ is represented by

$$\mathbf{A}^{\dagger} = \mathbf{A}^{T}(\mathbf{A}\mathbf{A}^{T})^{-1}$$

- See Appendixes A for more information.

- In order to take into account the performance index, $\mathbf{z}$:

$$\mathbf{x} = \mathbf{A}^{\dagger}\mathbf{b} + (\mathbf{1} - \mathbf{A}^{\dagger}\mathbf{A})\mathbf{z}$$

- Now apply the above for the velocity equation:

$$\boldsymbol{\theta} = \mathbf{J}\dot{\boldsymbol{\theta}}$$

- which becomes:

$$\dot{\boldsymbol{\theta}} = \mathbf{J}^{\dagger}\boldsymbol{\theta} + (\mathbf{1} - \mathbf{J}^{\dagger}\mathbf{J})\mathbf{z}$$

The Redundant Serial Manipulators

The Undetermined Linear Case–Application

- Assume $\dot{\boldsymbol{\theta}}$ to be a solution, therefore

$$\dot{\boldsymbol{\theta}} + \boldsymbol{\mathcal{N}}\dot{\boldsymbol{\theta}}_0$$

is also a solution.
- $\dot{\mathbf{t}}_0$ can be specified so as to satisfy an additional constraint to the problem. Assume the following

$$g(\dot{\boldsymbol{\theta}}) = \frac{1}{2}(\dot{\boldsymbol{\theta}} - \dot{\boldsymbol{\theta}}_0)^T(\dot{\boldsymbol{\theta}} - \dot{\boldsymbol{\theta}}_0)$$

- We use the Lagrange multiplier

$$g(\dot{\boldsymbol{\theta}}, \boldsymbol{\lambda}) = \frac{1}{2}(\dot{\boldsymbol{\theta}} - \dot{\boldsymbol{\theta}}_0)^T(\dot{\boldsymbol{\theta}} - \dot{\boldsymbol{\theta}}_0) + \boldsymbol{\lambda}^T(\dot{\mathbf{t}} - \mathbf{J}\dot{\boldsymbol{\theta}})$$

$$\dot{\boldsymbol{\theta}} = \mathbf{J}^T\boldsymbol{\lambda} + \dot{\boldsymbol{\theta}}_0 \quad \dot{\mathbf{t}} = \mathbf{J}\dot{\boldsymbol{\theta}}$$

$$\boldsymbol{\lambda} = (\mathbf{J}\mathbf{J}^T)^{-1}(\dot{\mathbf{t}} - \mathbf{J}\dot{\boldsymbol{\theta}}_0)$$

$$\dot{\boldsymbol{\theta}} = \mathbf{J}^\dagger\dot{\mathbf{t}} + (\mathbf{1} - \mathbf{J}^\dagger\mathbf{J})\dot{\boldsymbol{\theta}}_0$$

The Redundant Serial Manipulators

The Undetermined Linear Case–Application, Cont'd

- Generally, the following is considered for the objective function:

$$\dot{\boldsymbol{\theta}}_0 = k_0 \left(\frac{\partial \omega(\mathbf{q})}{\partial \mathbf{q}} \right)^T$$

- Typical objective:
  1. Manipulibility
  2. Distance from mechanism joint limits
  3. Distance from an obstacle
- Now, let's have a look to your homewrok

The Redundant Serial Manipulators

The Undetermined Linear Case–Application, Cont'd

- Generally, the following is considered for the objective function:

$$\dot{\boldsymbol{\theta}}_0 = k_0 \left(\frac{\partial \omega(\mathbf{q})}{\partial \mathbf{q}} \right)^T$$

$$\omega(\boldsymbol{\theta}) = \sqrt{\det\left(\mathbf{J}(\mathbf{q})\mathbf{J}^T(\mathbf{q})\right)}$$

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$$\omega(\boldsymbol{\theta}) = -\frac{1}{2} \sum_{i=1}^{n} \left(\frac{\theta_i - \overline{\theta}_i}{\theta_{iM} - \theta_{im}} \right)^2$$

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$$\omega(\boldsymbol{\theta}) = \min_{\mathbf{p},\mathbf{o}} \|\mathbf{p}(\boldsymbol{\theta}) - \mathbf{o}\|$$

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