

**Advanced Robotics  
Analytical Approaches  
for the IKP of Serial Manipulators  
By: M. Tale Masouleh  
Deadline: 1392/02/10  
Responsible TA: Amir-Hossein Karimi**

### **Problem 1 (To be Solved by TA)**

An experimental six-revolute robot is shown in an arbitrary posture in Fig 3.1.

- Produce the table with the Denavit-Hartenberge parameters of the manipulator;
- Find all arm inverse kinematic solutions for the positioning of point  $C$ , the center of the spherical wrist;

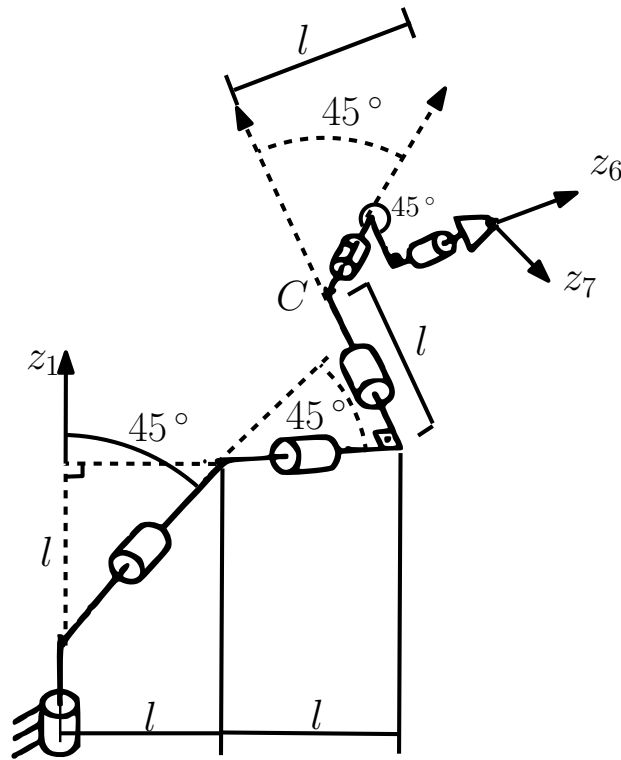


Figure 3.1: Arbitrary posture of an experimental six-revolute manipulator.

- Find all wrist inverse kinematic solutions for the orientations of the EE. In how many postures can this manipulator produce the same EE pose?

## Problem 2

For the manipulator with the Denavit-Hartenberge parameters represented in Table 3.1:

- Find all the inverse kinematic solutions for the positioning of point  $C$ , the operation point;
- Solve the forward kinematic problem of this manipulator for an arbitrary configuration;

Table 3.1: Denavit-Hartenberge parameters of the manipulator in problem 2.

$i$	$a_i$	$b_i$	$\alpha_i$	$\theta_i$
1	0	$b_1$	$\frac{\pi}{2}$	$\frac{\pi}{2}$
2	0	1	$\frac{\pi}{2}$	$\theta_2$
3	1	1	0	$\theta_3$

- Find the Jacobian matrix of the manipulator at any configuration;
- Using the Robotic Toolbox for an arbitrary operation point trajectories, solve the inverse kinematic problem to obtain some solution for the joint trajectories. Verify the results via computing the forward kinematic solutions and comparing them with each other;
- Compare the trajectories obtained by the Robotic Toolbox with the results of the second item in this problem.

### Problem 3

An experimental six-revolute robot is depicted in an arbitrary posture in Table 3.2.

- For the defined trajectory find the joint space trajectories by solving the IKP, using Robotic Toolbox.

$$x = 0.5 \sin t + 0.5, \quad y = 0.5 \cos t + 0.5, \quad z = t, \quad \text{for } t \in [0, \frac{\pi}{4}] \quad (3.1)$$

- Based on the above solutions, solve the FKP using the D-H parameters and compared the solution by the one obtained using the Robotic Toolbox.

### Problem 4

Attached to this HW is the data sheet of the Motoman UP50N robot. Based on this sheet, produce a table of Denavit-Hartenberg parameters that describe the geometry of the robot. In the figures, all dimensions are indicated

Table 3.2: Denavit-Hartenberge parameters of an experimental six-revolute manipulator.

$i$	$a_i$	$b_i$	$\alpha_i$	$\theta_i$
1	1	1	$\frac{\pi}{4}$	$\theta_1$
2	0	1	$\frac{\pi}{2}$	$\theta_2$
3	1	0	0	$\theta_3$
4	0	0	$-\frac{\pi}{2}$	$\theta_4$
5	0	0	$\frac{\pi}{2}$	$\theta_5$
6	0	1	0	$\theta_6$

in mm. In order to speed up the marking, use the - and + signs associated with the joints to define the  $Z_i$ -axes unambiguously.

- For the defined trajectory find the joint space trajectories by solving the IKP, using Robotic Toolbox.

$$x = 50 \sin t + 800, \quad y = 50 \cos t, \quad z = 1000, \quad \text{for } t \in [0, \frac{\pi}{2}] \quad (3.2)$$

- Based on the above solutions, solve the FKP using the D-H parameters and compared the solution by the one obtained using the Robotic Toolbox.

## Problem 5

The reasoning applied in Section 4.4.2 of the book in order to solve the orientation problem of the IKP is based on the following sequence:

$$\theta_4 \longrightarrow \theta_5 \longrightarrow \theta_6 \quad (3.3)$$

In this question, you should solve the same problem as above but for the following sequence:

$$\theta_4 \longrightarrow \theta_6 \longrightarrow \theta_5 \quad (3.4)$$

More specifically, you should find the following:

- First you should start from the fact that:

$$\mathbf{R} = \mathbf{Q}_4 \mathbf{Q}_5 \mathbf{Q}_6 = \mathbf{Q}_3^T \mathbf{Q}_2^T \mathbf{Q}_1^T \mathbf{Q} \quad (3.5)$$

- Show that the  $[\mathbf{e}_5]_4$  can be made equivalent to the last column of  $\mathbf{Q}_4$  and  $[\mathbf{e}_6]_4$  is the last column of  $\mathbf{R}\mathbf{Q}_6^T$ ;
- Find a univariate expression with respect to  $\theta_4$ , using the fact that the projection of  $\mathbf{e}_5$  into  $\mathbf{e}_6$  is a constant value;
- For  $\theta_4$ , you should find  $W, X, Y$  with respect to the known values

$$W c_4 + X s_4 = \quad (3.6)$$

where  $c_4 = \cos \theta_4$  and  $s_4 = \sin \theta_4$ ;

- For  $\theta_6$ :

$$\cos \theta_6 = \frac{r_{12} \sin \alpha_4 \sin \theta_4 - \dots}{\sin \alpha_5 \cos \alpha_6} \quad (3.7)$$

$$\sin \theta_6 = \frac{r_{11} \sin \alpha_4 \sin \theta_4 - \dots}{\sin \alpha_5} \quad (3.8)$$

From the above, it can be inferred that when  $\cos \alpha_6 = 0$  the above fails to provide a solution for  $\theta_6$ . Find an alternative to circumvent the latter problem;

- For  $\theta_5$ :

$$\cos \theta_5 = \frac{r_{31} \cos \theta_6 - \dots}{\sin \alpha_5 \cos \alpha_6} \quad (3.9)$$

$$\sin \theta_5 = \frac{\cos \alpha_4 \cos \alpha_5 \dots}{\sin \alpha_5} \quad (3.10)$$

To have a more consistent answer, consider:

$$\mathbf{R} = \mathbf{Q}_4 \mathbf{Q}_5 \mathbf{Q}_6 = \mathbf{Q}_3^T \mathbf{Q}_2^T \mathbf{Q}_1^T \mathbf{Q} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \quad (3.11)$$

$$\mathbf{Q}_5 \mathbf{Q}_6 = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \quad (3.12)$$

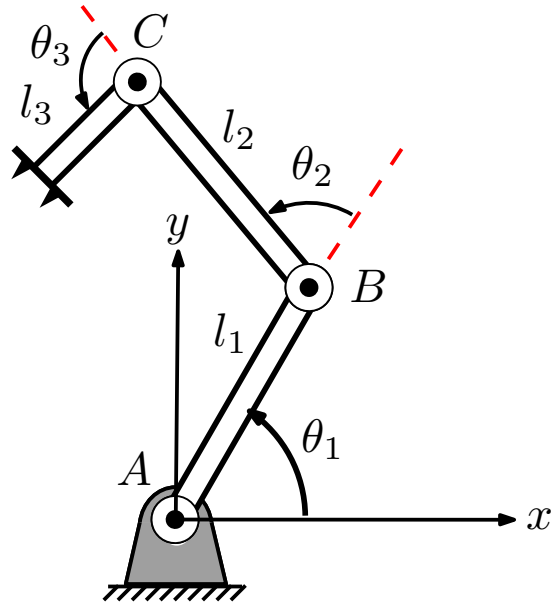


Figure 3.2: 3-DOF serial robot.

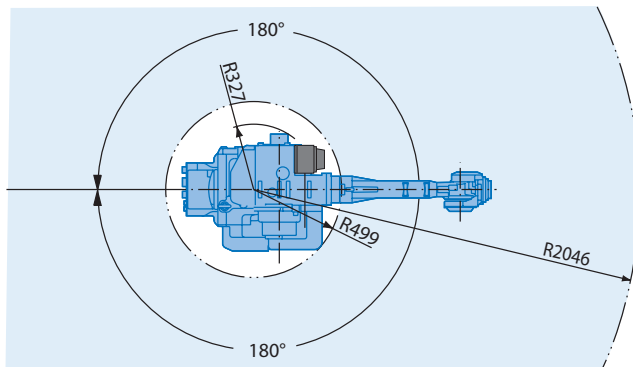
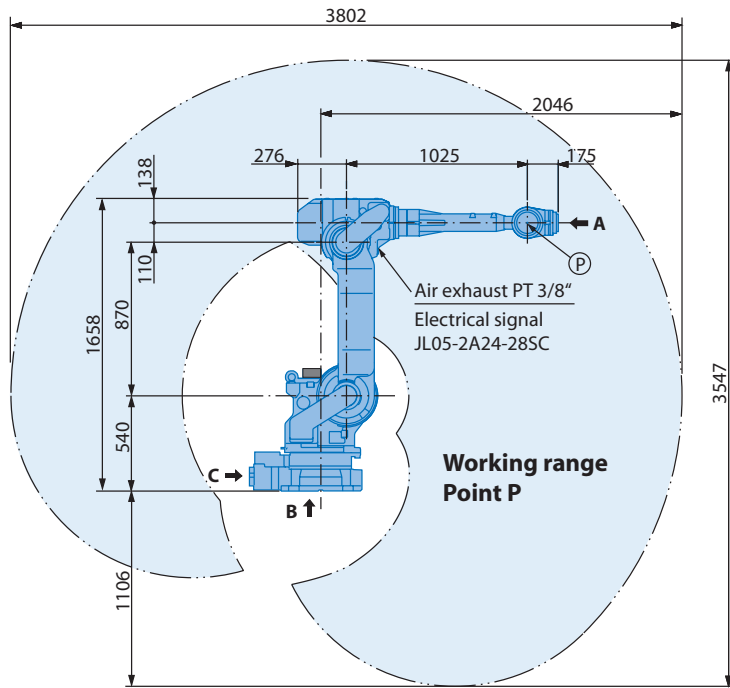
## Problem 6

Obtain the dexterity of a 3-DOF planar serial manipulator, depicted in Fig. 3.2, by:

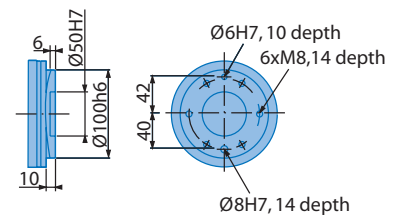
1. Derive the Jacobian;
2. Derive the dexterity of the mechanism;
3. For  $\theta_2 = \frac{3\pi}{4}$ ,  $l_1 = 1$  and  $l_2 = \frac{\sqrt{2}}{2}$  for  $l_3 = [\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1]$  plot (2D) the dexterity with respect to  $\theta_3$ ;
4. For  $l_1 = 1$ ,  $l_2 = 1$  and  $l_3 = 1$  plot (3D) the dexterity with respect to  $\theta_2$  and  $\theta_3$ ;
5. Plot the global dexterity with respect to  $\theta_2$  and  $\theta_3$ .

## Problem 7

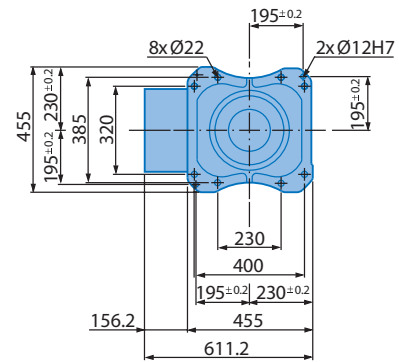
Fig. 3.2 represents schematically a RRR serial manipulator. Provide an optimal set of design parameters, i.e.,  $l_1$ ,  $l_2$  and  $l_3$ , for this serial manipulator, which compromises the best possible global dexterity, obtained in item 5 of Problem 6, according to the guidelines of the Differential Evolution (DE) optimization algorithm. The required optimization software codes for DE, developed by Mohammad Hossein Saadatzi, are available here. In case of any question, do not hesitate to contact Morteza Daneshmand: [mzdaneshmand@ieee.org](mailto:mzdaneshmand@ieee.org).



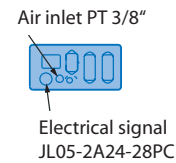
View A



View B



View C



**Notice**

- These dimensions are for reference purposes only. Please request detailed drawings for all design/engineering requirements, at [www.motoman.de](http://www.motoman.de).
- All dimensions in mm



Technical data may be subject to change without previous notice, UP50N, C-05-2008

Specifications UP50N

Axes	Maximum motion range [°]	Maximum speed [°/sec.]	Allowable moment [Nm]	Allowable moment of inertia [kg/m <sup>2</sup> ]	Controlled axes	
S	±180	170	-	-	Max. payload [kg]	6
L	+135/-90	170	-	-	Repet. pos. accuracy [mm]	50
U	+280/-160	170	-	-	Max. working range [mm]	±0,07
R	±360	250	196	13	Temperature [°C]	R = 2046
B	±125	250	196	13	Humidity [%]	0 bis +45
T	±360	350	127	5,5	Weight [kg]	20 - 80
					Power supply, average [KVA]	550
						5,0



**Headquarters**  
 Kammerfeldstraße 1  
 D-85391 Allershausen  
 Fon 0049-81 66-90-0  
 Fax 0049-81 66-90-103  
 info@motoman.de  
 www.motoman.de

**Training centre and sales office Frankfurt**  
 Hauptstraße 185  
 D-65760 Eschborn  
 Fon 0049-61 96-777 25-0  
 Fax 0049-61 96-777 25-39  
 info@motoman.de  
 www.motoman.de