Title of your Project
A Template for the Master Presentation

Full name of you and your Supervisor and Co-supervisor

University of Tehran

Human and Robot Interaction Laboratory
(TaarLab)

Seminar Course by Mehdi Tale Masouleh

Kinematics of Parallel Mechanaisms

Introduction

- Indices for Dimensionally Homogeneous Jacobian
  1. Dexterity (Condition number)
  2. Manipulability
- Indices for Dimensionally Nonhomogeneous Jacobian
  1. Multiple EE point velocities
  2. Normalized Jacobian
- Maximum Kinematic Sensitivity
  1. Point-displacement KS $\sigma_{p_{c,f}}$
  2. Rotation KS $\sigma_{r_{c,f}}$
- Jacobian representation

Thanks to Ehsan Faghih for the animations

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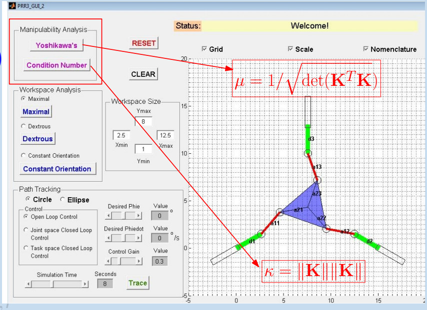

GUI by Priyanshu Agarwal

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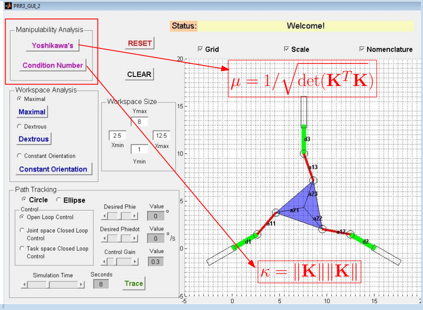

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$$\sigma_{p_{c,f}} \equiv \max_{\|\boldsymbol{\rho}\|_c=1} \| \mathbf{p} \|_f$$

$$\sigma_{r_{c,f}} \equiv \max_{\|\boldsymbol{\rho}\|_c=1} \| \boldsymbol{\phi} \|_f$$

$$\|\rho\|_2 = \sqrt{\rho_1^2 + \dots \rho_n^2}$$

$$\|\rho\|_\infty = \max\{\rho_1, \dots, \rho_n\}$$

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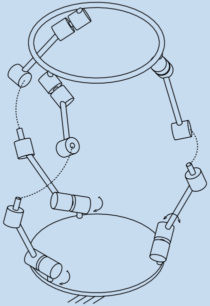

$$\sigma_{r_{c,f}} \equiv \max_{\|\boldsymbol{\rho}\|_c=1} \| \phi \|_f$$

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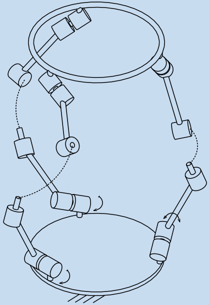

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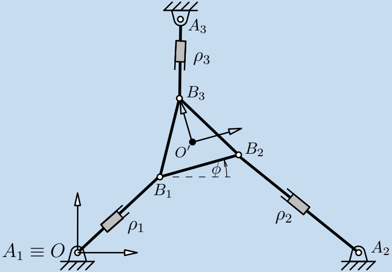

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KS for $c = \infty$ and $f = \{2, \infty\}$

General Formulation $\longrightarrow$ 3-R$\underline{\text{P}}$R

- $\|\boldsymbol{\rho}\|_\infty \leq 1$
- $\|\mathbf{Kx}\|_\infty \leq 1 \equiv$ zonotope in $\mathbb{R}^6$
- Vertices can be found by $[\mathbf{K}^T \quad -\mathbf{K}^T]\Delta\mathbf{x} \preceq \mathbf{1}_{12}$
- KS is a local index, $\mathbf{K}$ is known
- All the vertices
- $\sigma_{p_{\infty,\infty}} = \max(\max_{i=1,\dots,4} |x_i|, \max_{i=1,\dots,4} |y_i|)$
- $\sigma_{p_{\infty,2}} = \sqrt{\max_{i=1,\dots,4} (x_i^2 + y_i^2)}$
- $\sigma_{r_{\infty,\infty}} = \sigma_{r_{\infty,2}} = \max_{i=1,\dots,4} |\phi_i|$

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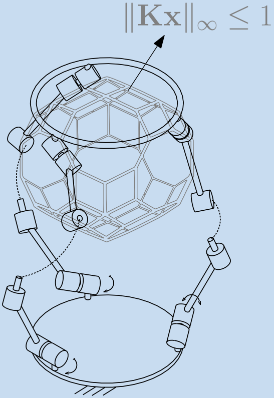

KS for $c = \infty$ and $f = \{2, \infty\}$

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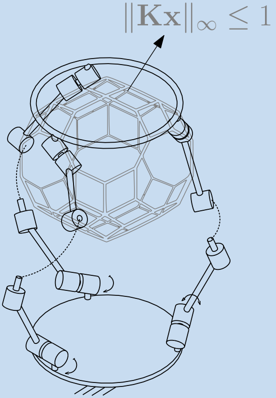

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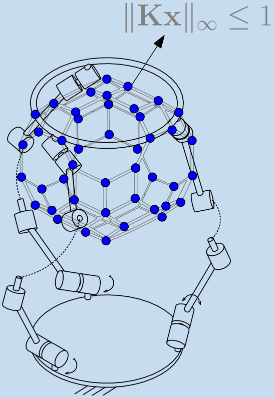

KS for $c = \infty$ and $f = \{2, \infty\}$

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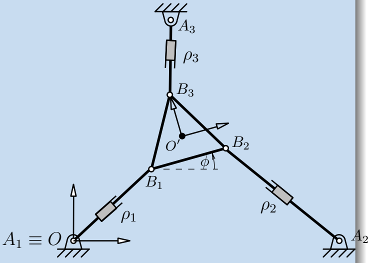

KS for $c = \infty$ and $f = \{2, \infty\}$

General Formulation $\longrightarrow$ 3-R$\underline{\text{P}}$R

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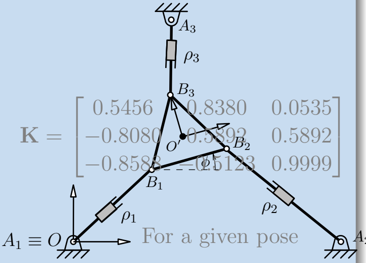
$$\mathbf{K} = \begin{bmatrix} 0.5456 & 0.8380 & 0.0535 \\ -0.8080 & 0.5892 & 0.5892 \\ -0.8588 & -0.5123 & 0.9999 \end{bmatrix}$$



For a given pose

KS for $c = \infty$ and $f = \{2, \infty\}$

General Formulation $\longrightarrow$ 3-RPR

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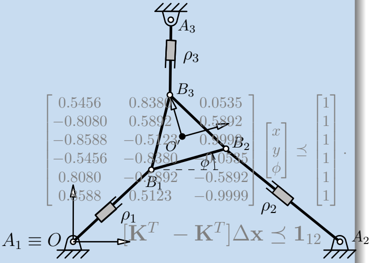

$$\begin{bmatrix} 0.5456 & 0.8380 & 0.0535 \\ -0.8080 & 0.5892 & 0.5892 \\ -0.8588 & -0.5123 & 0.9999 \\ -0.5456 & -0.8380 & 0.0535 \\ 0.8080 & -0.5892 & -0.5892 \\ 0.8588 & 0.5123 & -0.9999 \end{bmatrix} \begin{bmatrix} x \\ y \\ \phi \end{bmatrix} \preceq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

$$[\mathbf{K}^T \;\; -\mathbf{K}^T]\Delta\mathbf{x} \preceq \mathbf{1}_{12}$$

KS for $c = \infty$ and $f = \{2, \infty\}$

General Formulation $\longrightarrow$ 3-R$\underline{\text{P}}$R

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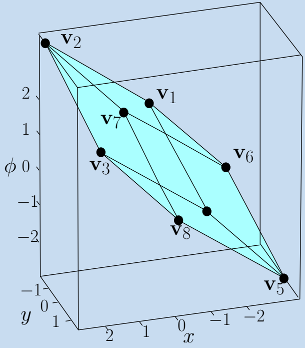

KS for $c = \infty$ and $f = \{2, \infty\}$

General Formulation ⟶ 3-RPR

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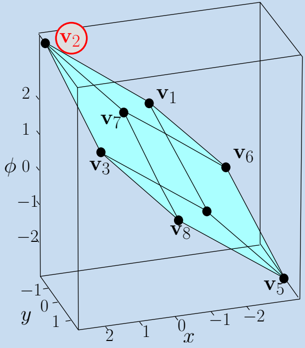

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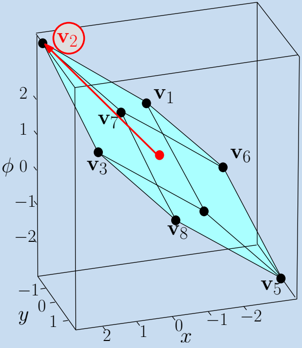

KS for $c = \infty$ and $f = \{2, \infty\}$

General Formulation $\longrightarrow$ 3-R$\underline{\text{P}}$R

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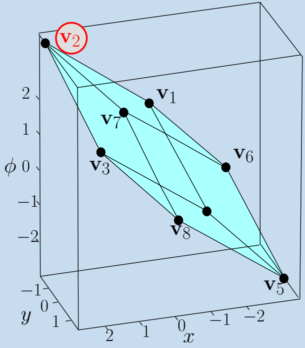
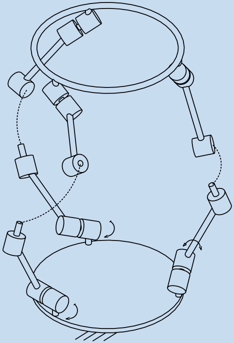


KS for $c = 2$ and $f = \{2, \infty\}$

General Formulation $\longrightarrow$ 3-R$\underline{\text{P}}$R

1. $\|\mathbf{Kx}\|_2 \leq 1 \equiv$ ellipsoid in $\mathbb{R}^6$
2. For point-displacement KS
3. For rotation KS
4. $\sigma_{p2,2}$ and $\sigma_{r2,2} \equiv$ semimajor
5. $\sigma_{p2,\infty}$ and $\sigma_{r2,\infty} \equiv$ an interval
6. A 3-R$\underline{\text{P}}$R



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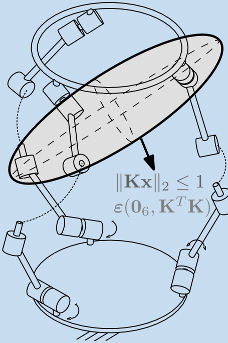

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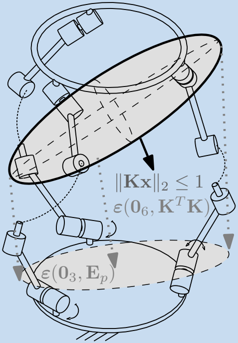

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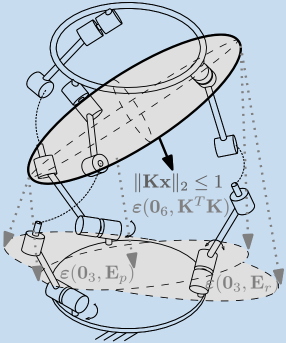

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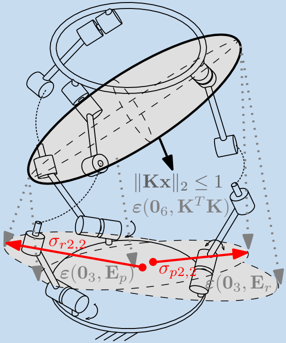

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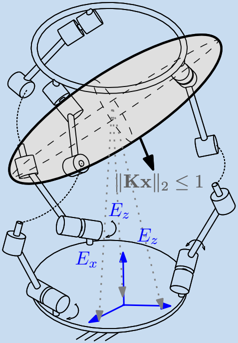

KS for $c = 2$ and $f = \{2, \infty\}$

General Formulation $\longrightarrow$ 3-RPR

1. $\|\mathbf{Kx}\|_2 \leq 1 \equiv$ ellipsoid in $\mathbb{R}^6$
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3. For rotation KS
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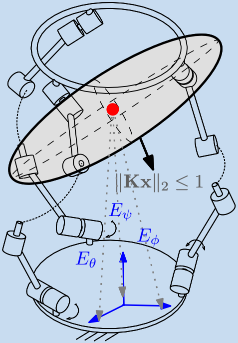
5. $\sigma_{p2,\infty}$ and $\sigma_{r2,\infty} \equiv$ an interval
6. A 3-RPR



KS for $c = 2$ and $f = \{2, \infty\}$

General Formulation ⟶ 3-RPR

1. $\|\mathbf{Kx}\|_2 \leq 1 \equiv$ ellipsoid in $\mathbb{R}^6$
2. For point-displacement KS
3. For rotation KS
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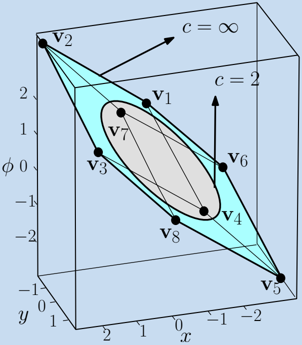

Comparison Between Different Variants of the KS

Some observations

1. $\sigma_{\infty,2} \geq \sigma_{\infty,\infty} \geq \sigma_{2,\infty}$
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4. KS is frame-invariant
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8. Which one should be considered ! $\Longrightarrow$ Redundant PMS

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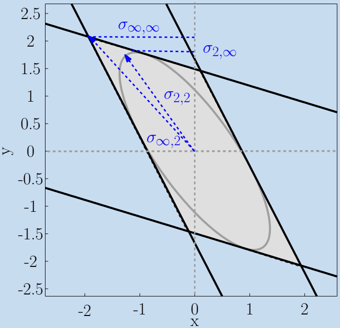

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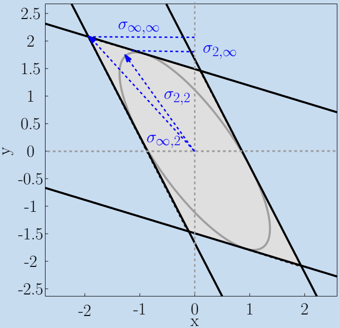

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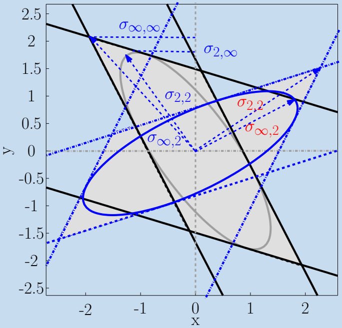

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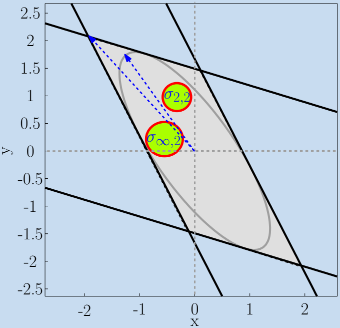

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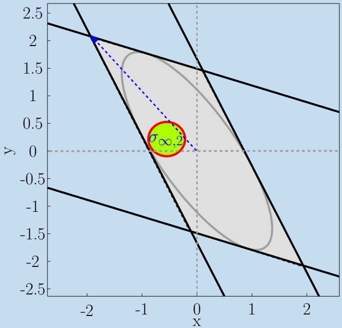

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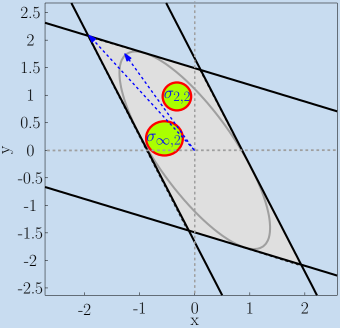
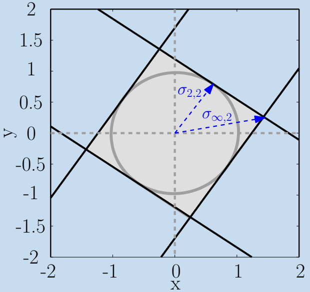
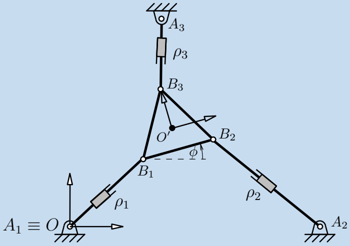


KS of Redundant PMs

Some conclusions





Non-redundant 3-DOF

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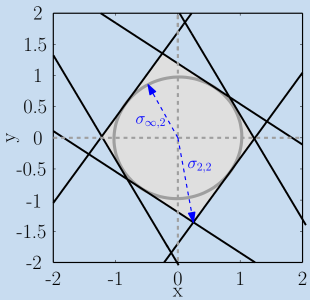

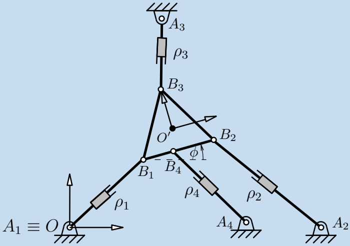

Redundant 3-DOF

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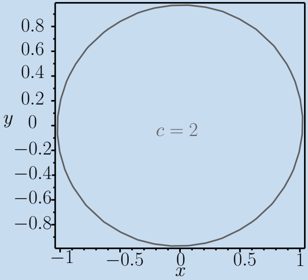

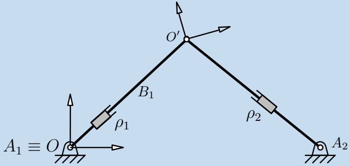

Non-redundant 2-DOF

KS of Redundant PMs

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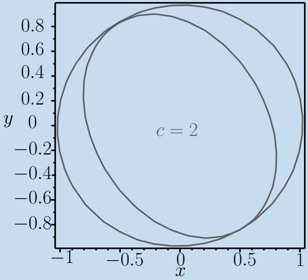

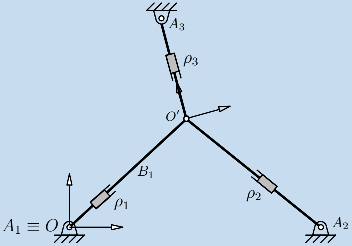

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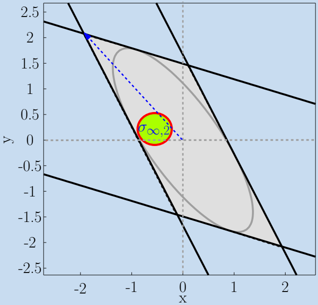

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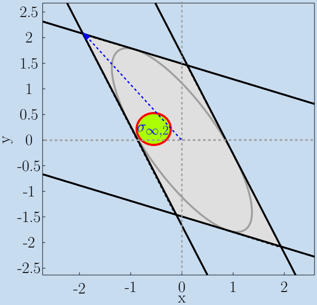

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Global Kinematic Sensitivity

From Local to Global KS

1. $\zeta_I = \dfrac{\int_W I dW}{\int_W dW}$
2. Not working in singularity !
3. Indices for optimization :

$$\sigma'_{r,2} = \frac{1}{1 + \sigma_{r,2}}$$

$$\sigma'_{p,2} = \frac{1}{1 + \sigma_{p,2}}$$

4. $0 \leq \sigma'_{r,2} \leq 1, \quad 0 \leq \sigma'_{p,2} \leq 1$
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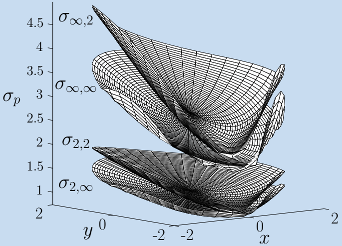

8-UPS

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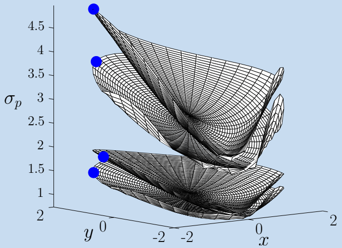

8-UPS

Conclusion

Summary for Kinematic Sensitivity

1. From some generalities to a case study : 3-R$\underline{P}$R
2. Analysing different norms for the constraint and function
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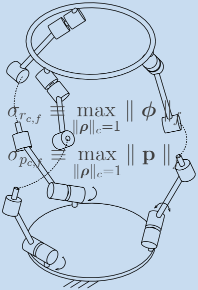

Ongoing and Future Work

1. Global Kinematic Sensitivity
2. Optimization

Conclusion

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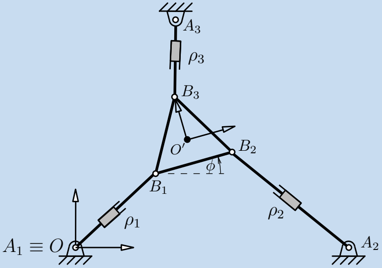

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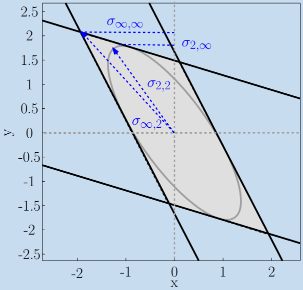

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University of Tehran

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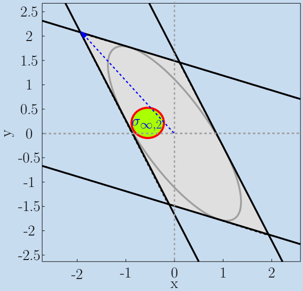

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TaarLab

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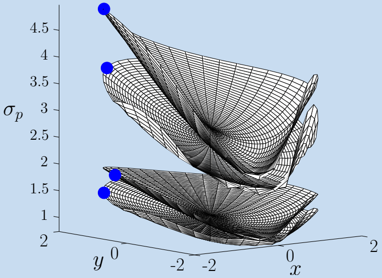

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Acknowledgments

Thanks to

Spiritual Sponsors

1. Natural Sciences and Engineering Research Council of Canada (NSERC)
2. Canada Research Chair program
3. Iran National Science Foundation (INSF) research grant.
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