Static and Strength of Materials

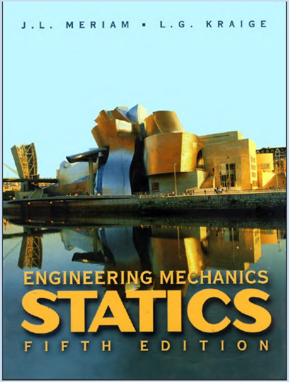
Mehdi Tale Masouleh

October 17, 2013

Overview

Our objective

- References
- Homework & Projects
- Exam
- How to reach me:
- TA



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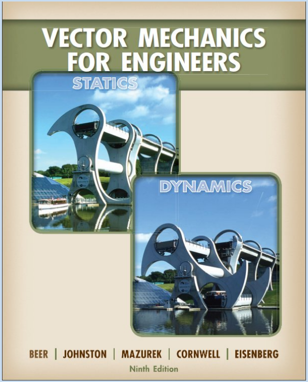

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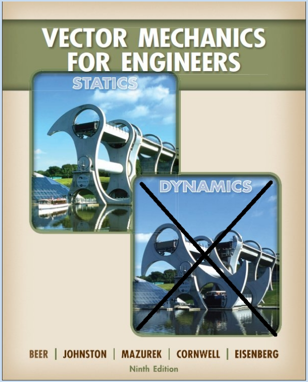

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1. Returning some problems per chapter
2. A GUI for analyzing 2D truss
3. A project based on Solid Works
4. Static balancing of Four-bar mechanisms

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1. One quiz per chapter
2. One final Exam

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- Scribble me at :
  m.t.masouleh@ut.ac.ir
  mehdi.tale.masouleh@gmail.com
- My office: A217 and B217
- My office phone number: 61118574
- Human and Robot Interaction Lab.

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Mostafa Saket

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Mahmood Ghafouri

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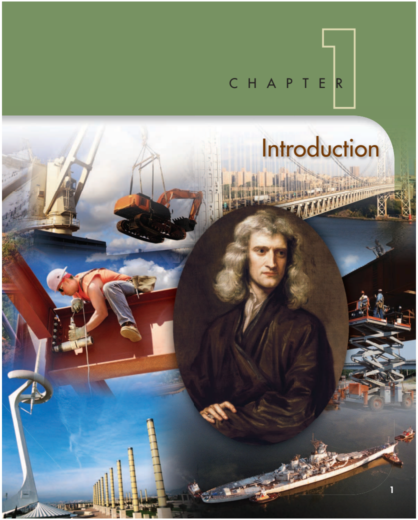
CHAPTER 1
Introduction
1

Introduction

- What is Mechanincs!?
- First in this filed:
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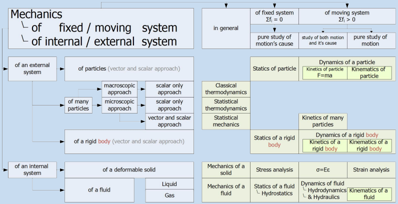

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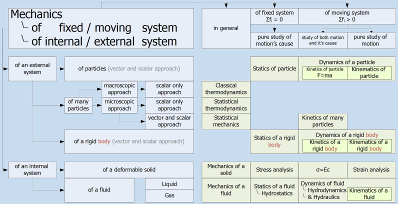

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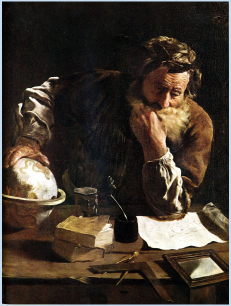

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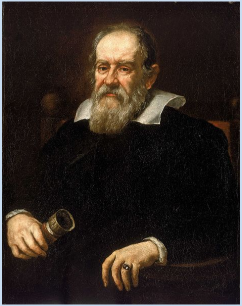

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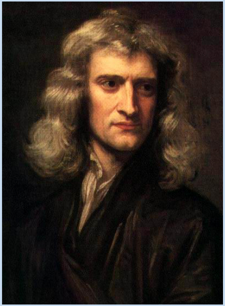

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Fundamental Concept in Mechanics

Five Concepts in Mechanics

- Space
- Time × In Dynamics ✓
- Mass ✓
- Force ✓
- A particle ✓
- Rigid body ✓

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- The geometric region occupied by bodies.
- Determined relative to some geometric reference system
- *Primary inertial system* or *astronomical frame of reference*.
- no translation and no rotation
- Newtonian laws $\longrightarrow$ the velocity involved in the system is less than the speed of light $\approx 300\,000$ Km/s

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- Quantities of the succession of event.
- Absolute quantity in Newtonian mechanics

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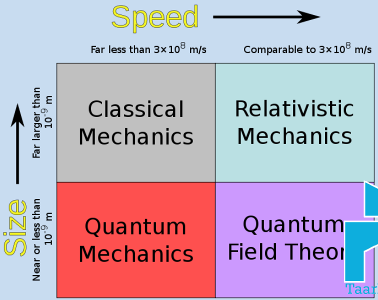

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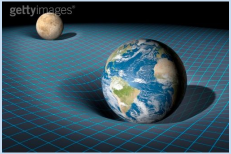
- Quantitative measure of the inertia or resistance to change in motion of a body.
- Quantity of matter in a body
- The property which gives rise to the gravitational attraction.



"Einstein proposed that

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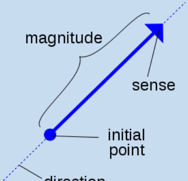
- The vector (!) action of one body on another ▸ Homework
- Tends to displace a body based on its direction and line of action.
- Magnitude, direction and point of application



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- A body with negligible dimensions

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- a body whose changes in shape are negligible compared with the
  1. overall dimensions of the body
  2. with the changes in the position of the body.

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Vector and Scalar

A review

- Mechanics is governed by two quantities:
  1. Scalar: a magnitude
  2. Vector: a magnitude+direction !

Controversial

Everything with direction and magnitude can be considered as vector! The parallelogram rule should be applicable.

Vector and Scalar

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- Both are Tensors!
- Zero-order tensor: Scalar
- first-order tensor: Vector
- Coming from *Tensor Product*

Tensor Product

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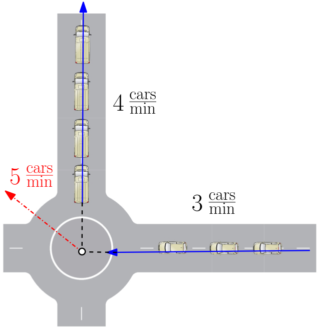

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Vector Operations

Review

- Scalar: Italic lowercase
- Notation: Vector lowercase and boldface type
- Addition:
  1. Triangle addition
  2. Parallelogram addition
  3. Commutative law
  4. Associative law
- Subtraction
- Scalar multiplication
- Unit vectors: **i**, **j** and **k**
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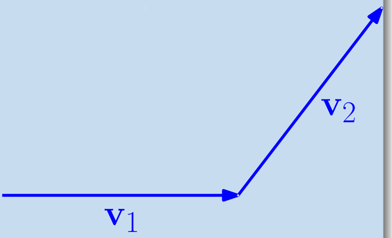

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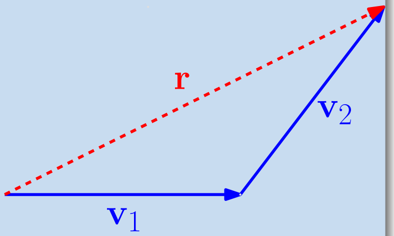

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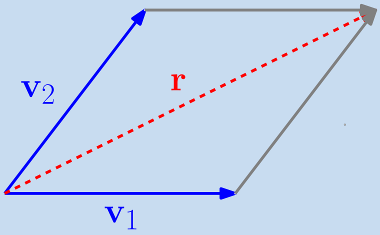

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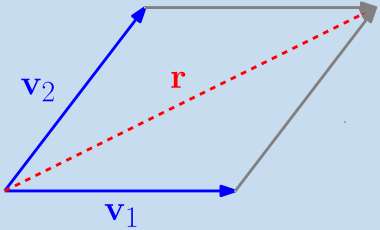

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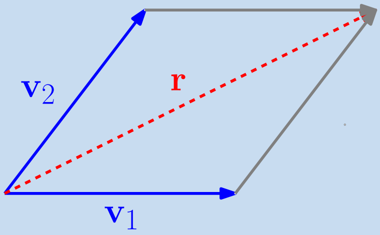

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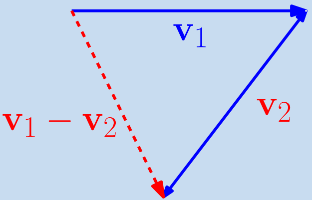

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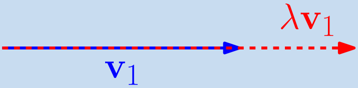

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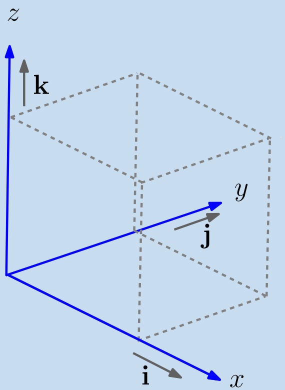

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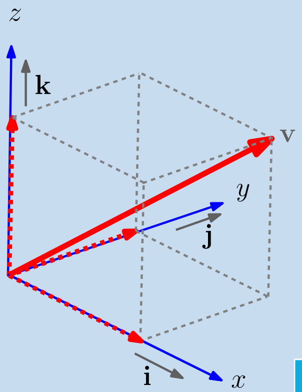

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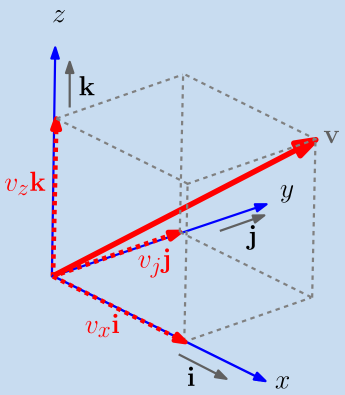

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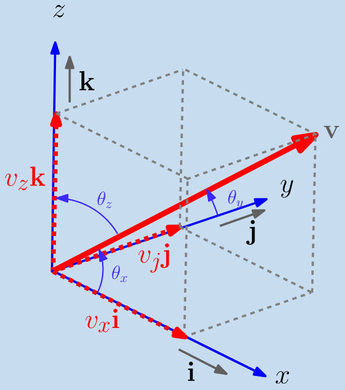

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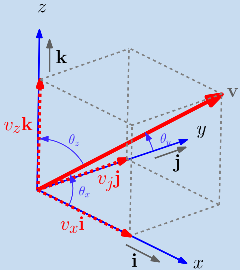

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$

$$l = \cos\theta_x,\ m = \cos\theta_y,\ n = \cos\theta$$

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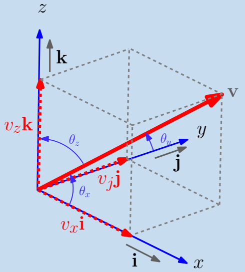

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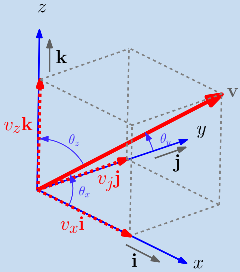

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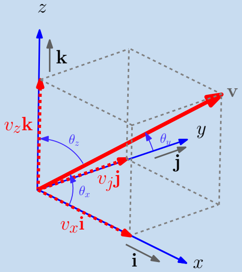

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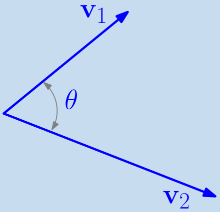
- A first-order scalar tensor product
- $\mathbf{v}_1 \cdot \mathbf{v}_2 = v_1 v_2 \cos\theta$
- $\cos\theta = \dfrac{\mathbf{v}_1 \cdot \mathbf{v}_2}{v_1 v_2}$
- $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$ then $\mathbf{v}_1 \perp \mathbf{v}_2$



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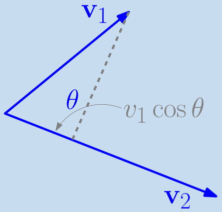

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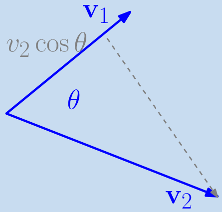

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Back

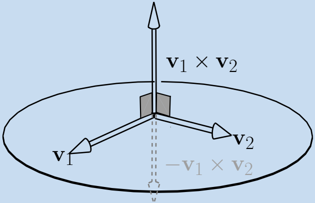
Review-Dot or Scalar Product

- A first-order scalar tensor product
- $\mathbf{v}_1 \cdot \mathbf{v}_2 = v_1 v_2 \cos\theta$
- $\cos\theta = \dfrac{\mathbf{v}_1 \cdot \mathbf{v}_2}{v_1 v_2}$
- $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$ then $\mathbf{v}_1 \perp \mathbf{v}_2$

Vector Operations

Review- Cross or Vector Product

- $|\mathbf{v}_1 \times \mathbf{v}_2| = v_1 v_2 \sin\theta$
- Triple scalar product
- Triple vector product
- Some useful relations in kinematics and Statics



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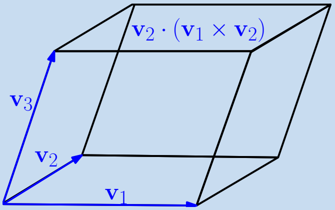

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$$\mathbf{v}_1 \times (\mathbf{v}_2 \times \mathbf{v}_3) = (\mathbf{v}_1 \cdot \mathbf{v}_3)\mathbf{v}_2 - (\mathbf{v}_1 \cdot \mathbf{v}_2)\mathbf{v}_3$$

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$$|\mathbf{a} \times \mathbf{b}| = \sqrt{(\mathbf{a}\cdot\mathbf{a})(\mathbf{b}\cdot\mathbf{b}) - (\mathbf{a}\cdot\mathbf{b})^2}$$

Vector Operations

Review- Cross or Vector Product

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$$(\mathbf{a}\times\mathbf{b})\times(\mathbf{c}\times\mathbf{d}) = (\mathbf{abd})\mathbf{c}-(\mathbf{abc})\mathbf{d}$$

Vector Operations

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$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$$

Application of Triple Vector Product

Nothing is useless!

- The Cartesian decomposition of **A**

$$\mathbf{A}_S = \frac{1}{2}(\mathbf{A} + \mathbf{A}^T), \qquad \mathbf{A}_{SS} = \frac{1}{2}(\mathbf{A} - \mathbf{A}^T)$$

- The *vector* of **A** is:

$$\mathbf{a} \times \mathbf{v} = \mathbf{A}_{SS}\mathbf{v}$$

- The *trace* of **a** is the sums of the eigenvalues of $\mathbf{A}_s$, are all real.
- We define the following:

$$\text{vect}(\mathbf{A}) = \mathbf{a} = \frac{1}{2}\begin{bmatrix} a_{32} - a_{23} \\ a_{13} - a_{31} \\ a_{21} - a_{12} \end{bmatrix} \qquad \text{tr}(\mathbf{A}) = a_{11} + a_{22} + a_{33}$$

Application of Triple Vector Product

Nothing is useless

- Show that

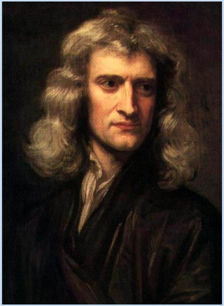
$$\mathrm{vect}(\mathbf{a}\mathbf{b}^T) = -\frac{1}{2}\mathbf{a}\times\mathbf{b}, \qquad \mathrm{tr}(\mathbf{a}^T\mathbf{b}) = \mathbf{a}^T\mathbf{b}$$

-

Newton's Laws of Motion

From your secondary

- First law ($\sum \mathbf{F} = \mathbf{0}$)
- Second law (Dynamic $\mathbf{F} = m\mathbf{a}$)
- Third law (Action & Re-action)



Units

International System of metric units (SI)

Quantity	Dimensional Symbol	Unit	Symbol
Mass	M	Kilogram	Kg
Length	L	meter	m
Time	T	second	s
Force	F	newton	N

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- Bureau International des Poids et Mesure
- an alloy of 90% platinum-10 % iridium

Force Systems-Section A: 2D Force System

Force

- In statics: Action of one body on another.
- Dynamics: Action which tends to cause acceleration $\mathbf{F} = m\mathbf{a}$
- A force is vector quantity ! direction and magnitude!
- Thus, we can use the parallelogram law! is it true!?
- Treat Force as Fixed vector in the case of cable Tension.
  1. Free vector: Movement without rotation
  2. sliding vector: Unique line of action but not unique point of application.
  3. Fixed vector. A fore on a non-rigid body.
- Action of a force as *External* and Internal: The relation of external and internal is the subject of *Strength of Materials*

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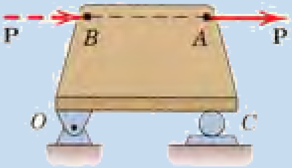
Force Systems-Section A: 2D Force System

Force and Principal of Transmissibility

- This channel us to regard Force as sliding vector
- Since we study the resultant of external forces
- Thus, we consider only the magnitude, direction and line of action (besides the point of action)



Force Systems-Section A: 2D Force System

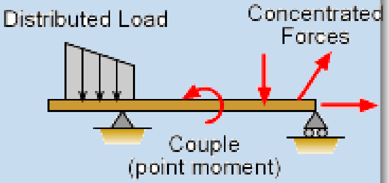
Force Classification

- Contact force: Physical contact
- Body force: position of a body within a force field: gravitational, electric: Your weight

Other classifications:

- Concentrated force
- Distributed force

The above classifications depends on the body under study.



Force Systems-Section A: 2D Force System

2D Force System

- We are not wasting our time for this part.
- Parallelogram law for concurrent at a point
- A special case: Two parallel forces
- Rectangular Components

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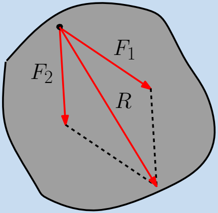

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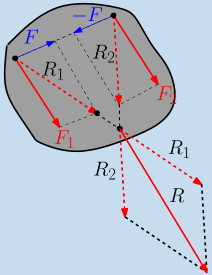

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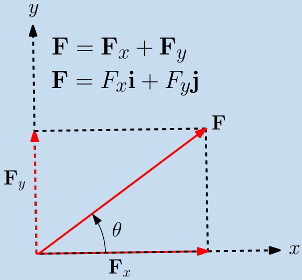
$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$
$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$



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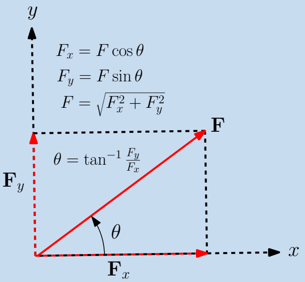
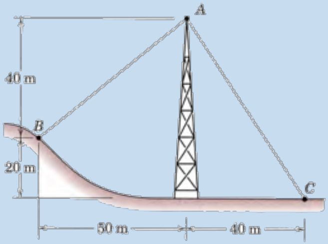
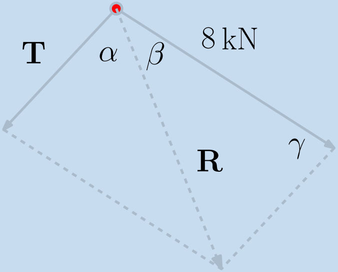


Force Systems-Section A: 2D Force System

Guy Cables





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Law of sines $\frac{\sin\beta}{T} = \frac{\sin\alpha}{8}$

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Force Systems-Section A: 2D Force System

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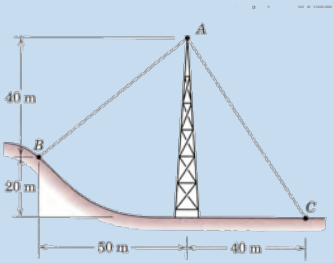

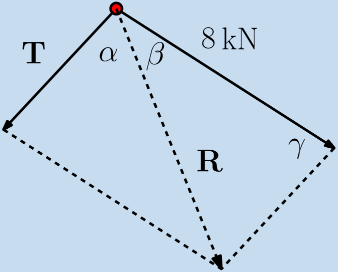

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TaarLab Human

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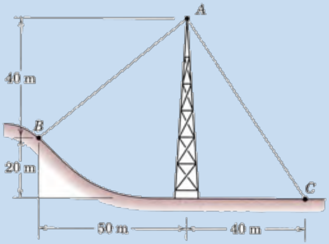

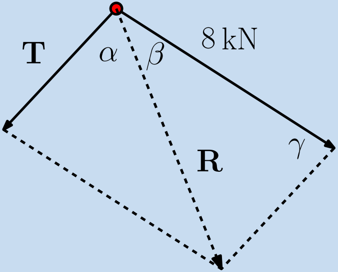

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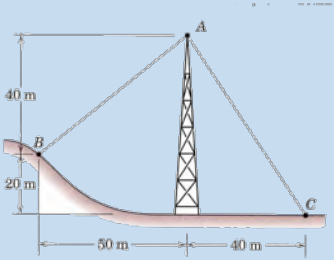

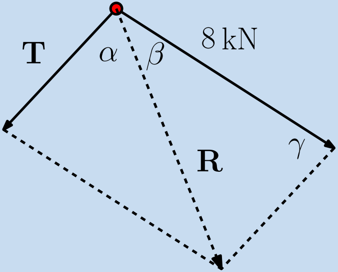

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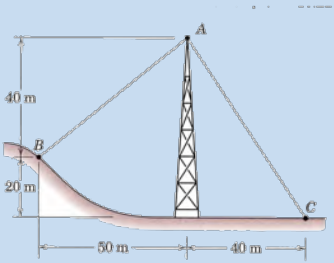

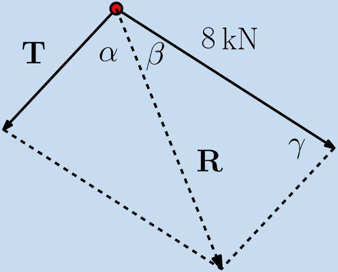

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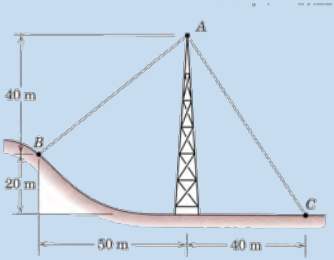

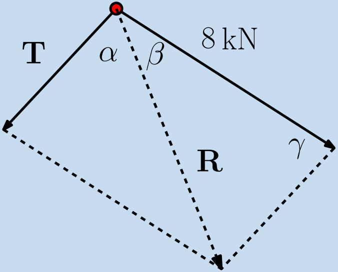

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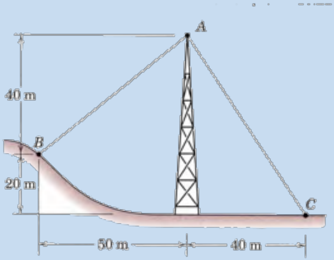

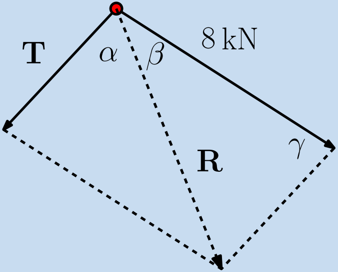

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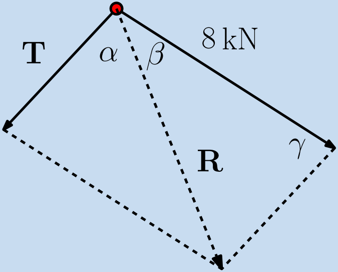

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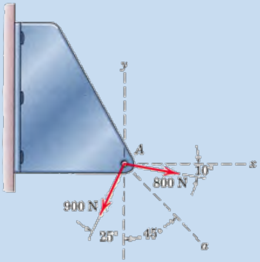
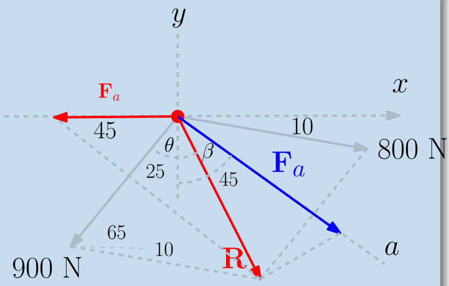
Force Systems-Section A: 2D Force System

The gusset plate





Law of cosines

$$R = \sqrt{900^2 + 800^2 - 2(800)(900)\cos 75^\circ}$$

Law of sines: $\frac{R}{\sin 75^\circ} = \frac{800}{\sin ...}$

$\beta = 180 - 45 - 65 - ...$

Law of sines: $\frac{f_a}{...} = \frac{R}{...}$

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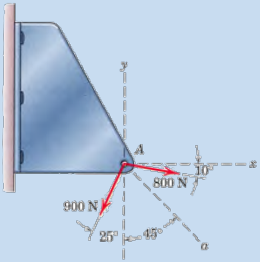

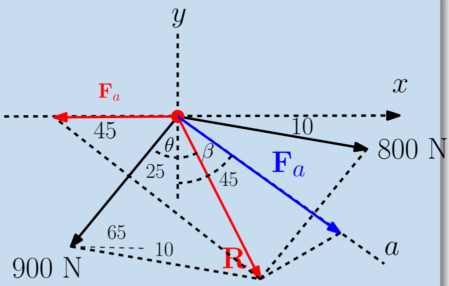

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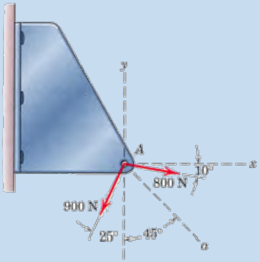

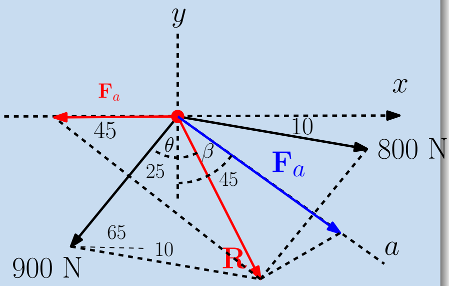

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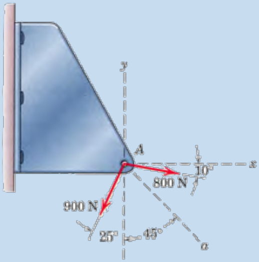

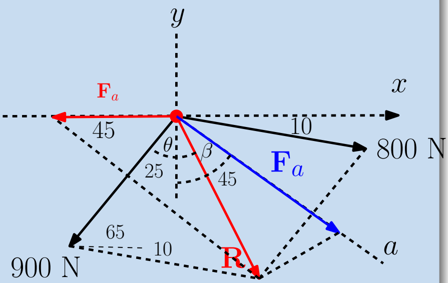

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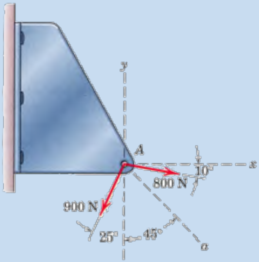

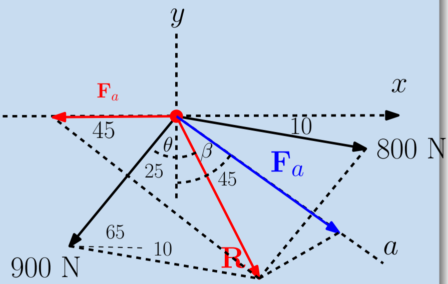

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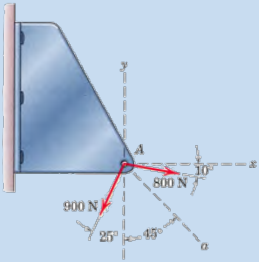

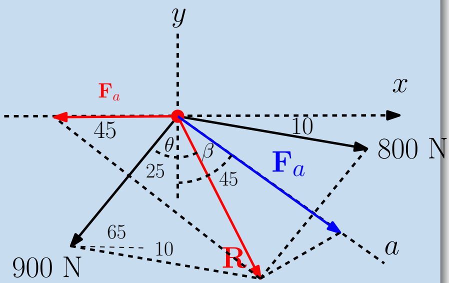

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Law of sines: $\dfrac{R}{\sin 75^\circ} = \dfrac{800}{\sin}$

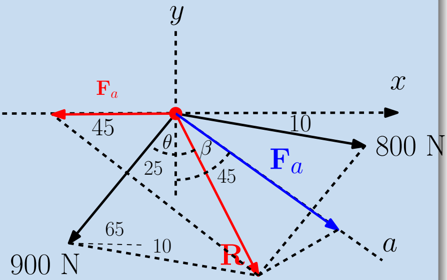
$\beta = 180 - 45 - 65 -$

Law of sines: $\dfrac{f_a}{} = \dfrac{R}{}$

Force Systems-Section A: 2D Force System

The gusset plate

Strength of Materials



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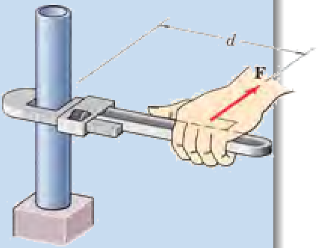
TaarLab Human

Moment

Moment

- Up to now: force as tendance to move in a given direction
- Now: tendency to rotated a body about an axis
- This axis (or line) neither intersects nor is parallel to the line of action of the forces.
- Magnitude of the moment: $M = Fd$
- The cross product $\mathbf{M} = \mathbf{r} \times \mathbf{F}$.
- Varignon's Theorem, This could be in your exam



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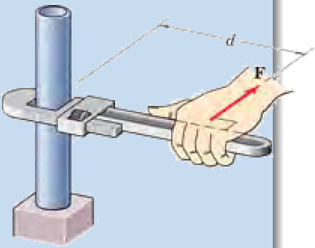

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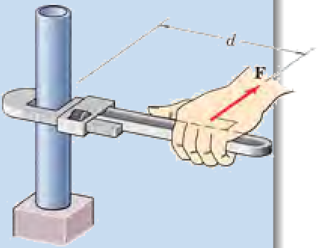

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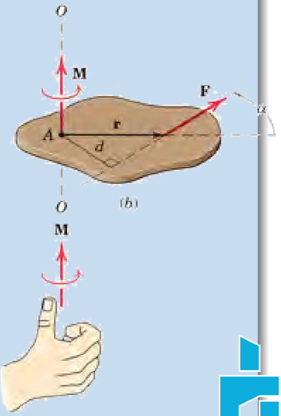

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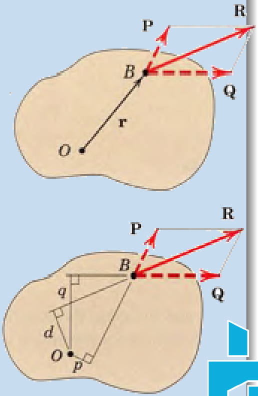
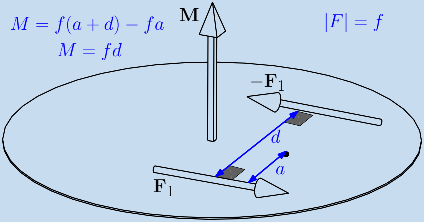
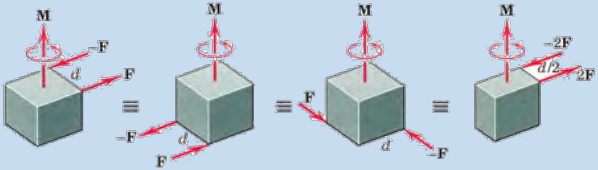


Couple

Couple

- Moment produced by two equal opposite and non-collinear forces
- The vector form could on your exam
- Force-Couple System





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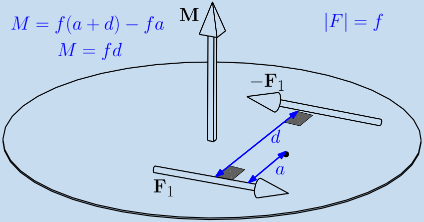
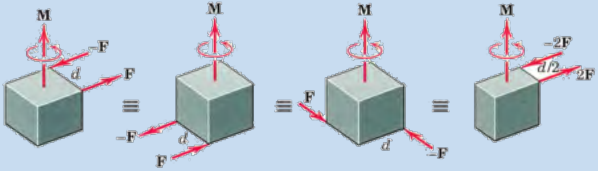




Some Hints for the Exam

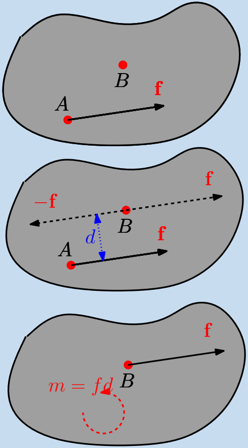
For instance

- Triple vector product (develop)
- The connection between Statics and Strength of Materials
- Application of law of sine;
- Application of law of cosine;
- Varginon's theorem;
- Derive the vector form of a couple;
- Different forms to represent a vector.

Force-Couple System

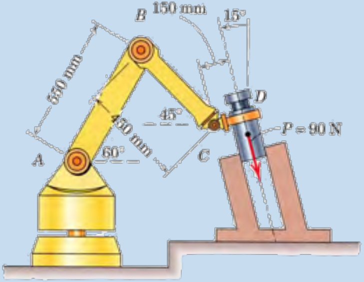
Force-Couple System

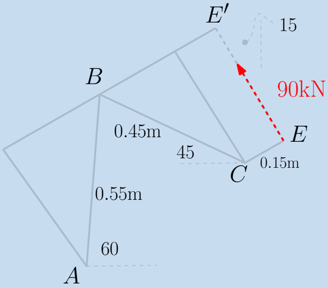
- Force-Couple System
- Replacement of force by an equivalent force-couple system.
- The reverse is also valid.
- They have any applications in mechanics.
- Of paramount importance in *Screw Theory*
- Thus it should be mastered.



Example-Moment

Force-Couple System





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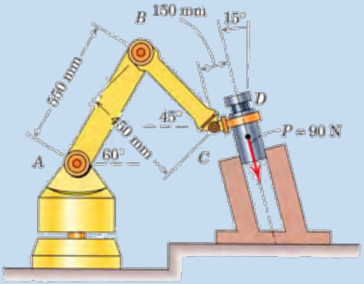

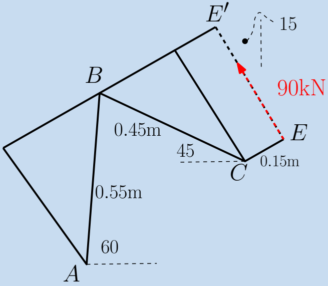

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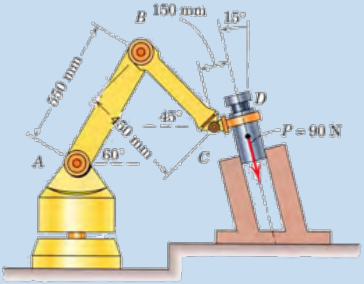
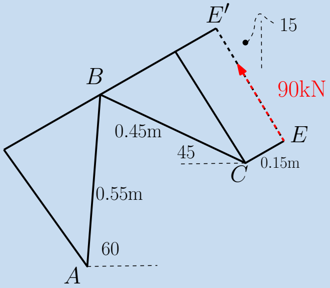
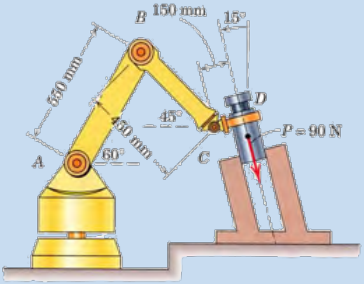


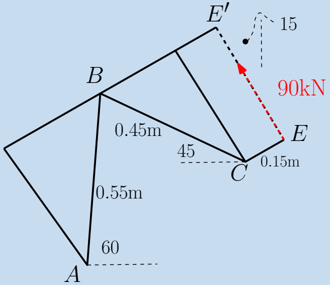


$$m_c = F(CE) = 90(0.15) = 13.5$$
N.m

Example-Moment

Force-Couple System

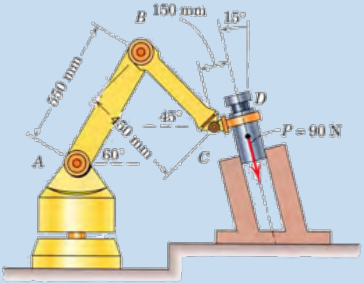


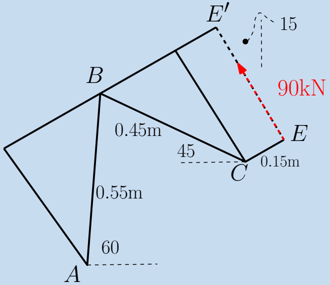


$$m_B = F(BE') = 90(0.15 + 0.45 \sin 45^\circ) = 33.8 \text{ N.m}$$

Example-Moment

Force-Couple System



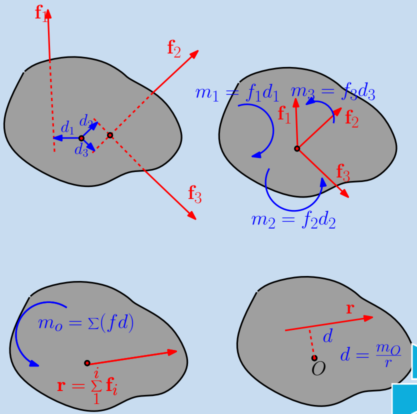


$$m_B = F(BE') = 90(0.15 + 0.45\sin 45^\circ + 0.55\sin 45^\circ) = 68.8 \text{ N.m}$$

Resultants-2D

Resultants

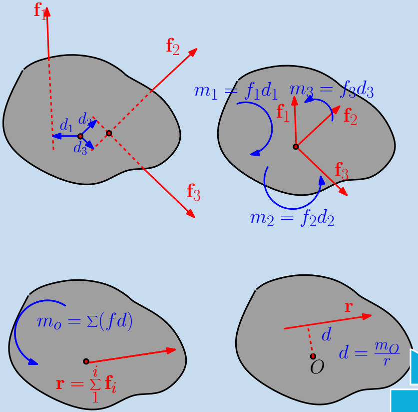
- Action of a group of system of forces
- Most mechanical systems deals with system of force
- Reduce to its simple form
- To the end of describing the action
- Definition of resultant: simplest force combination replacing the original force without alerting the external effect.



Resultants-2D

Resultants-Mathematic Formulation

- $\mathbf{r} = \sum \mathbf{f}_i$
- $r_x = \sum (f_x)_i$, $r_y = \sum (f_y)_i$
- $r = \sqrt{(\sum (f_x)_i)^2 + (\sum (f_y)_i)^2}$
- $\theta = \tan^{-1} \frac{r_y}{r_x} = \tan^{-1} \frac{\sum (f_y)_i}{\sum (f_x)_i}$
- $\mathbf{r} = \sum \mathbf{f}_i$
- $m_O = \sum m_i = \sum (f_i d)$
- $rd = m_O$



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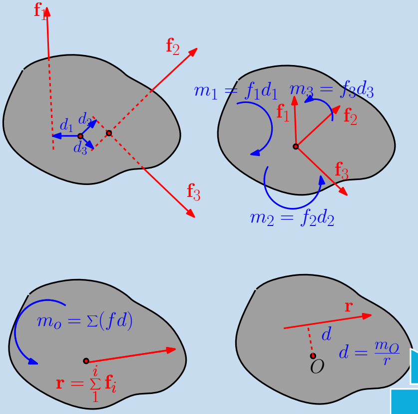

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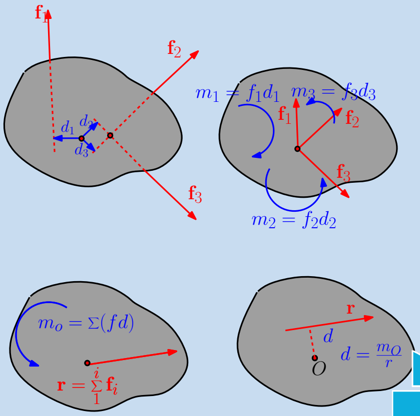

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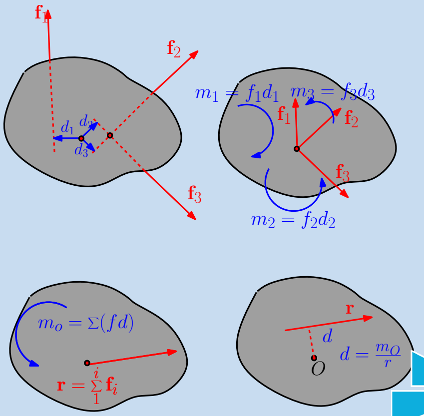

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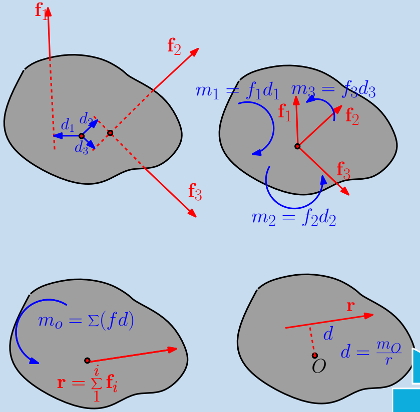

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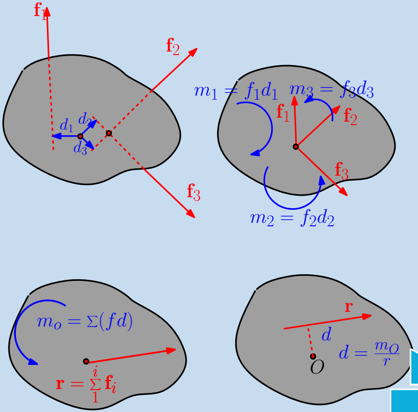

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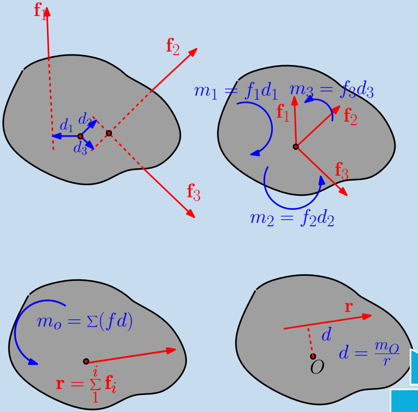

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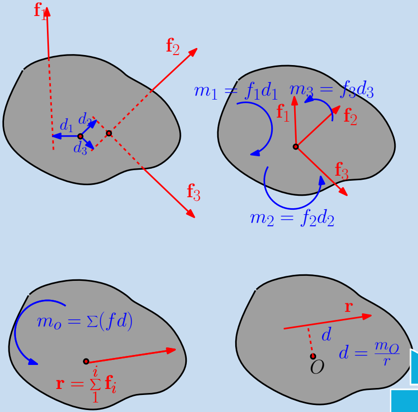
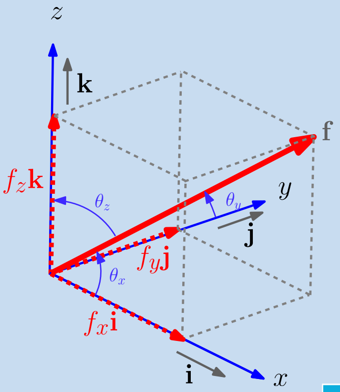


3D Forces Systems

Rectangular Components

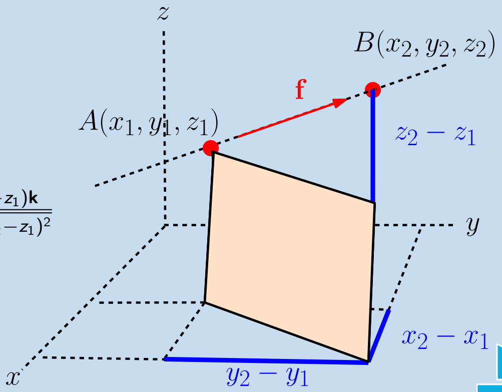
- $f_x = f\cos\theta_x$
  $f_y = f\cos\theta_y$
  $f_z = f\cos\theta_z$
- $f = \sqrt{f_x^2 + f_y^2 + f_z^2}$
  $\mathbf{f} = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$
  $\mathbf{f} = f(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z)$
- $\mathbf{n}_F = l\mathbf{i} + m\mathbf{j} + n\mathbf{k}$
  $\mathbf{f} = F\mathbf{n}_F$



3D Forces Systems-Rectangular Components

Two points on the line of action of the force

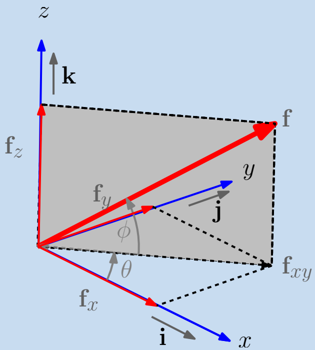
- $\mathbf{f} = f\mathbf{n}_f = f\frac{\vec{AB}}{\|AB\|}$
- $\mathbf{f} = f\frac{(x_2-x_1)\mathbf{i}+(y_2-y_1)\mathbf{j}+(z_2-z_1)\mathbf{k}}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$
- 5-DOF can be controlled



3D Forces Systems-Rectangular Components

Specification by Two Angles

- $f_{xy} = f \cos\phi$
- $f_z = f \sin\phi$
- $f_x = f_{xy} \cos\theta = f \cos\phi \cos\theta$
- $f_y = f_{xy} \sin\theta = f \cos\phi \sin\theta$
- Dot product ▸ Review
- Angle between two vectors ▸ Review



3D Moment and Couples

Moments in 3D

- $\mathbf{m}_O = \mathbf{r} \times \mathbf{f}$
- Right-hand rule
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- Moment about an arbitrary axis

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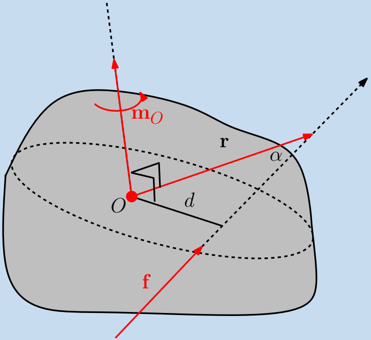

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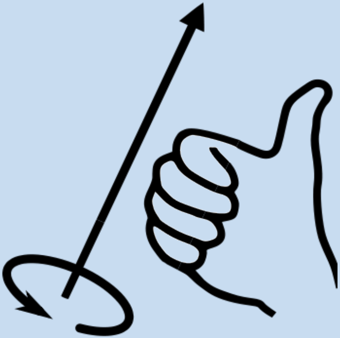

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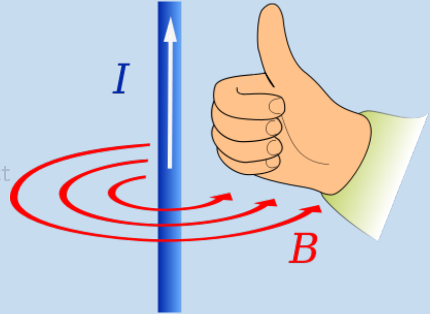

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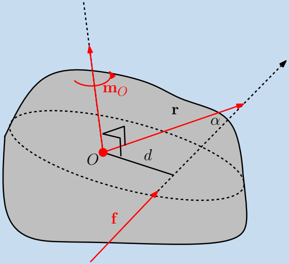
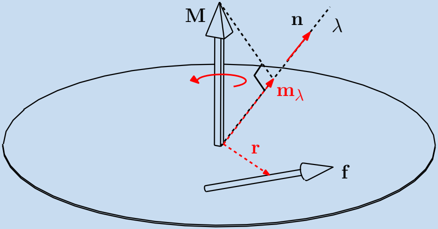


$$\mathbf{m}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ f_x & f_y & f_z \end{vmatrix}$$

3D Moment and Couples

Moments in 3D



- $\mathbf{m}_O = \mathbf{r} \times \mathbf{f}$
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- Moment about an arbitrary axis

$$\mathbf{m}_\lambda = ((\mathbf{r} \times \mathbf{f}) \cdot \mathbf{n})\mathbf{n}$$

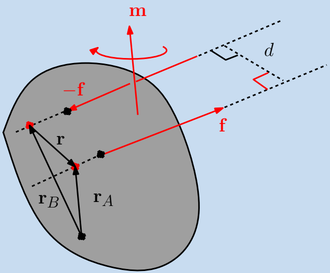
$$\mathbf{m}_O = \begin{vmatrix} r_x & r_y & r_z \\ f_x & f_y & f_z \\ \alpha & \beta & \gamma \end{vmatrix}$$

where α, β and γ stand for the direction cosine of the unite vector **n**

3D Moment and Couples

Couples in 3D

- Extended easily from the 2D
- $\mathbf{m} = \mathbf{r}_A \times \mathbf{f} + \mathbf{r}_b \times (-\mathbf{F})$
- Then,
  $\mathbf{m} = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{f} = \mathbf{r} \times \mathbf{f}$
- A very good example
- Equivalent force-couple system



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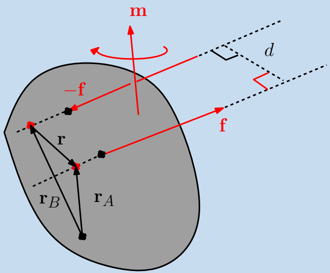

3D Moment and Couples

Couples in 3D

- Extended easily from the 2D
- $\mathbf{m} = \mathbf{r}_A \times \mathbf{f} + \mathbf{r}_b \times (-\mathbf{F})$
- Then,
  $\mathbf{m} = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{f} = \mathbf{r} \times \mathbf{f}$
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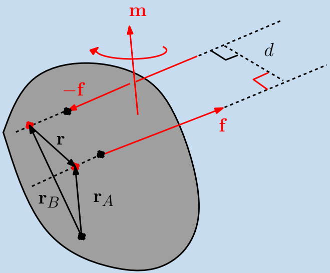

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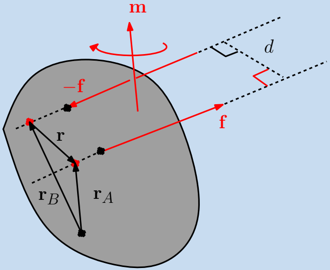

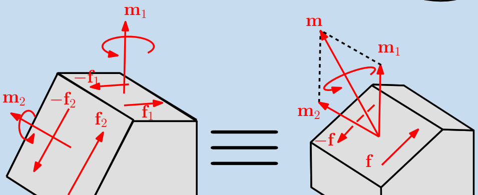

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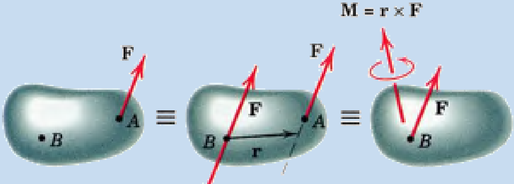
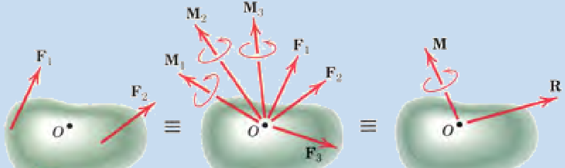


Resultants in 3D

The State of the Art

- The concept remains the same, Review Eqs. (2/20) and (2/21)
- Three special cases: (Important for exam)
  1. Concurrent forces: Only Eq. (2/20)
  2. Parallel forces: Sample Problem 2/14
  3. Coplanar forces: Article 2/6.

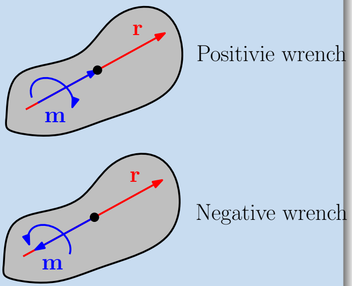
$$\mathbf{r} = \sum \mathbf{f}_i$$
$$\mathbf{m} = \sum \mathbf{m}_i$$
$$r = \sqrt{(\sum f_x)^2 + (\sum f_y)^2 + (\sum f_z)^2}$$
$$m = \sqrt{(m_x)^2 + (m_y)^2 + (m_z)^2}$$



Wrench Resultants

Basic Concepts

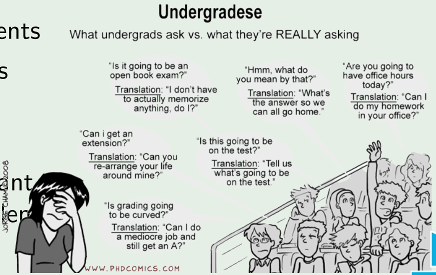
- When the resultant couple vector is parallel to the resultant force
- Every force and couple system can be reduced to a wrench
- There is a duality between kinematics and statics.
- This duality is governed by Wrench (Statics) and Twist (Kinematics)



Some Hints for the Exam

Concepts

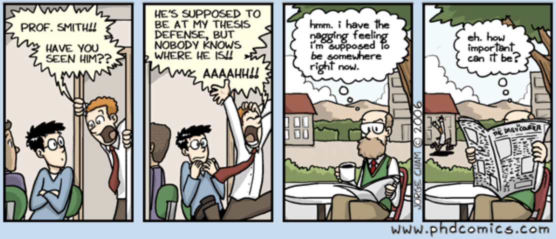
- Concept of force-couple system, 3D and 2D.
- Concept of resultant, 3D and 2D
- Concept of rectangular components
- Two methods to represent forces
- Concept of a representing a moment along an axis.
- Three special case for 3D resultant (More concerns on Sample problem 2/14)
- Twist and wrench concept
- Duality of statics and kinematics



Some Hints for the Exam

Prepare yourself for Section A, 2D

- 2D Force systems Rectangular Components: 2/13, 2/16 and 2/23
- 2D Force systems Moments: 2/33, 2/37, 2/35 and 2/42
- 2D Force systems Couple: 2/60, 2/65 and 2/68
- 2D Force systems, Resultants: 2/76, 2/77, 2/84 and 2/86

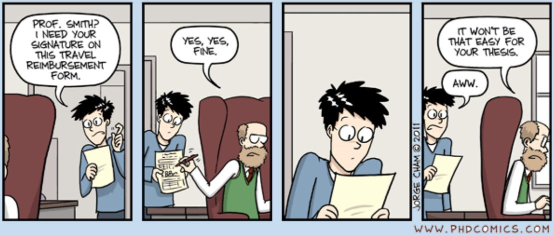


I will remember your exam day

Some Hints for the Exam

Prepare yourself for Section B, 3D

- 3D Force systems Rectangular Components:
  2/99, 2/105 and 2/107
- 3D Force systems Moments and Couples:
  2/117, 2/130 and 2/151
- 3D Force systems, Resultants:
  2/135, 2/144, 2/145 and 2/147
- Review for Chapter 2: 2/157, 2/161, 2/162 and 2/164



Matlab Workshop

Matlab by Mr. Molaei, Tuesday, 09/27/2011, 10:30 AM

- Programming
- Functions
- Plot
- To the end of:
  1. Get ready for your assignments and projects
  2. Filling the gaps that we are not able to fill in this course

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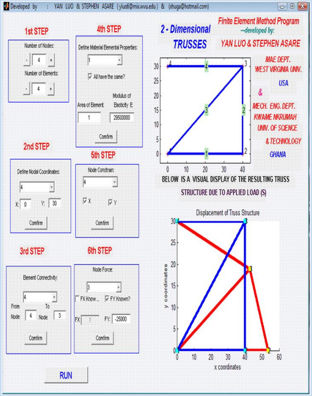

Tensor Product

Back

Tensor & Tensor Prodcut

- A multilinear transformation defined over an underlying finite dimensional vector space, $\mathcal{V}$.
- Zero-order Tensors: $\mathcal{T}^0$: isomorphic to scalar field. Linear transformation from $\mathcal{T}^0$ to$\mathcal{T}^0$:

$$\alpha[\cdot] : \quad \mathcal{T}^0 \longmapsto \mathcal{T}^0 \text{ meaning that } \beta \leftrightarrow \alpha[\beta] = \alpha\beta \tag{1}$$

- First-order Tensors , $\mathcal{T}^1$: Isomorphic to the underlying vector space $\mathcal{V}$. One has:
  - $\mathcal{T}^0 \longmapsto \mathcal{T}^1$ meaning that $\mathbf{a}[\alpha] = \alpha\mathbf{a}$
  - $\mathcal{T}^1 \longmapsto \mathcal{T}^0$ meaning that $\mathbf{a}[\mathbf{b}] = \mathbf{a} \cdot \mathbf{b}$