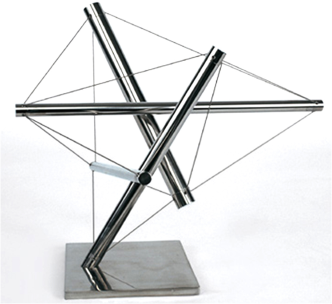
Static and Strength of Materials-Chapter 3-Equilibrium

Mehdi Tale Masouleh



October 17, 2013

Introductions

Introductions

- A new definition: Statics deals with the description of the force conditions necessary and sufficient to maintain the equilibrium of engineering structures.
- Form your secondary:

$$\mathbf{r} = \sum \mathbf{f}_i = 0 \qquad \mathbf{m} = \sum \mathbf{m}_i = 0$$

- Both are necessary and sufficient
- All physical bodies are 3D
- In some cases we can treat them as 2D
- When forces lie on single plane
- Can be projected onto a single plane

The same as Chapter 2:

1. Equilibrium in 2D
2. Equilibrium in 3D

Introductions

Introductions

- A new definition: Statics deals with the description of the force conditions necessary and sufficient to maintain the equilibrium of engineering structures.
- Form your secondary:

$$\mathbf{r} = \sum \mathbf{f}_i = 0 \qquad \mathbf{m} = \sum \mathbf{m}_i = 0$$

- Both are necessary and sufficient
- All physical bodies are 3D
- In some cases we can treat them as 2D
- When forces lie on single plane
- Can be projected onto a single plane

The same as Chapter 2:

1. Equilibrium in 2D
2. Equilibrium in 3D

Equilibrium in 2D

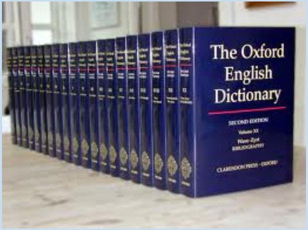
Definition of Mechanical System

- By J. Meriam: A body or group of bodies which can be conceptually isolated form all other bodies
- By Concise Oxford Dictionary: Complex whole, set of connected things or parts, organized body of material or immaterial things
- I will give you some definitions. Worth to be read.

Equilibrium in 2D

Definition of Mechanical System

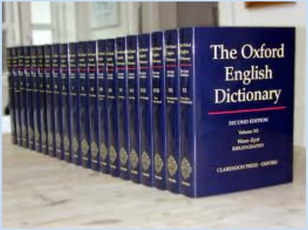
- By J. Meriam: A body or group of bodies which can be conceptually isolated form all other bodies
- By Concise Oxford Dictionary: Complex whole, set of connected things or parts, organized body of material or immaterial things
- I will give you some definitions. Worth to be read.



Equilibrium in 2D

Definition of Mechanical System

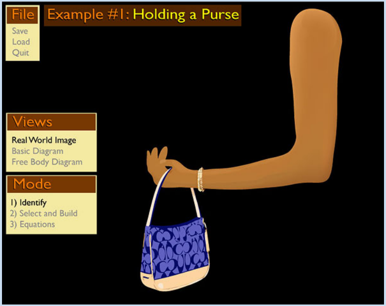
- By J. Meriam: A body or group of bodies which can be conceptually isolated form all other bodies
- By Concise Oxford Dictionary: Complex whole, set of connected things or parts, organized body of material or immaterial things
- I will give you some definitions. Worth to be read.



Equilibrium in 2D

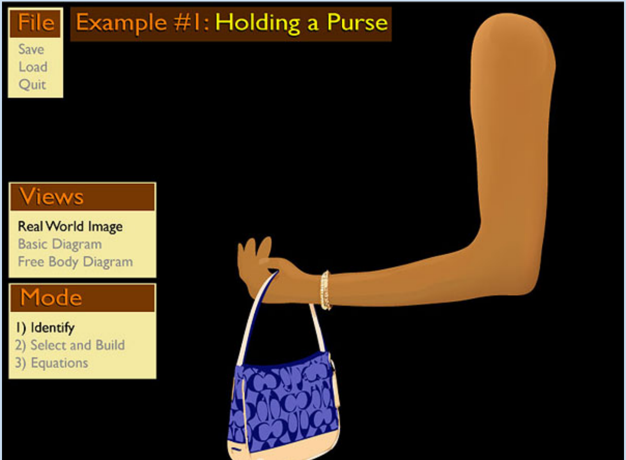
Free-body Diagram

- The free-body diagram is the most important single step i the solution of problems in mechanics.
- a diagrammatic representation of the isolated system treated as single body
- Only after such a diagram has been carefully draw should the equilibrium equations be written



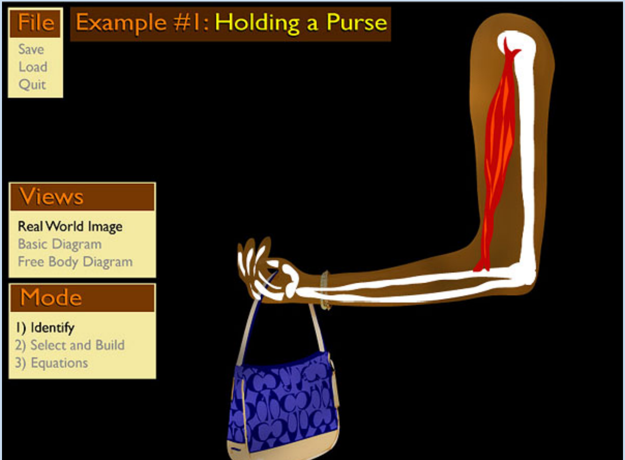
Equilibrium in 2D

Free-body Diagram, Holding a Purse



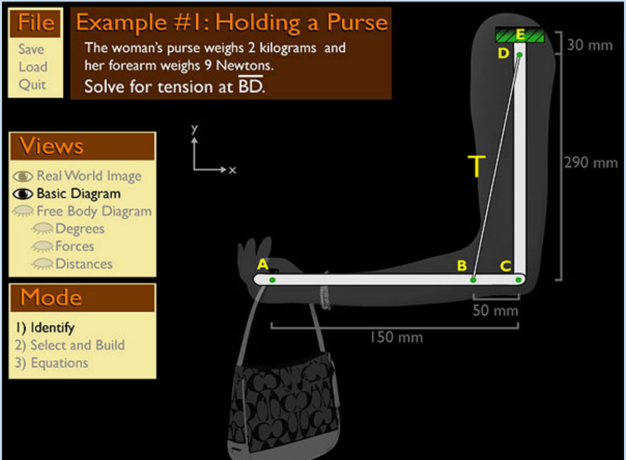
Equilibrium in 2D

Free-body Diagram, Holding a Purse



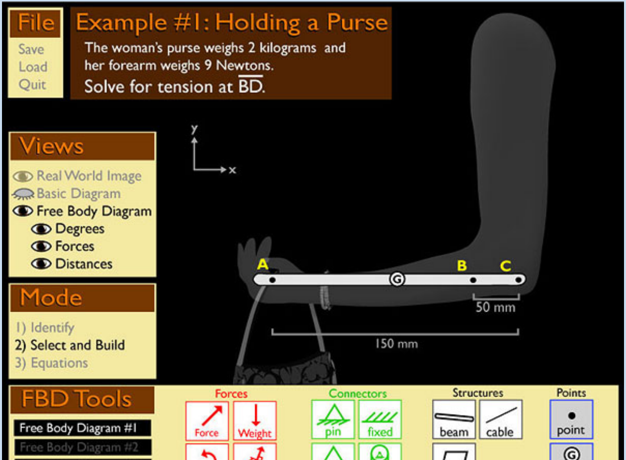
Equilibrium in 2D

Free-body Diagram, Holding a Purse



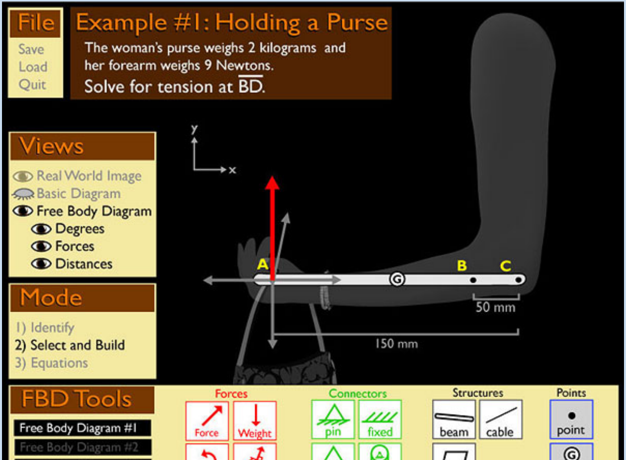
Equilibrium in 2D

Free-body Diagram, Holding a Purse



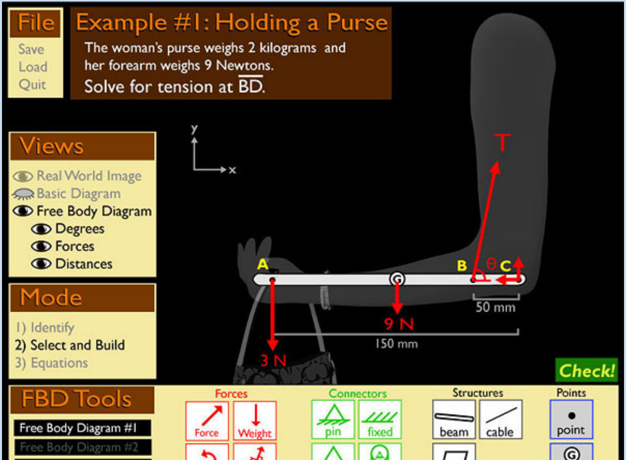
Equilibrium in 2D

Free-body Diagram, Holding a Purse



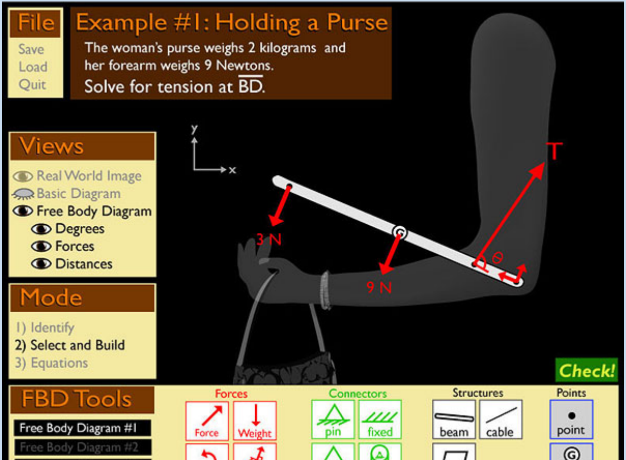
Equilibrium in 2D

Free-body Diagram, Holding a Purse



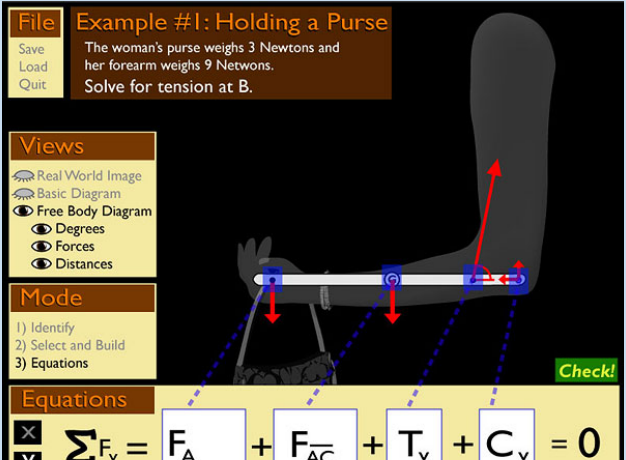
Equilibrium in 2D

Free-body Diagram, Holding a Purse



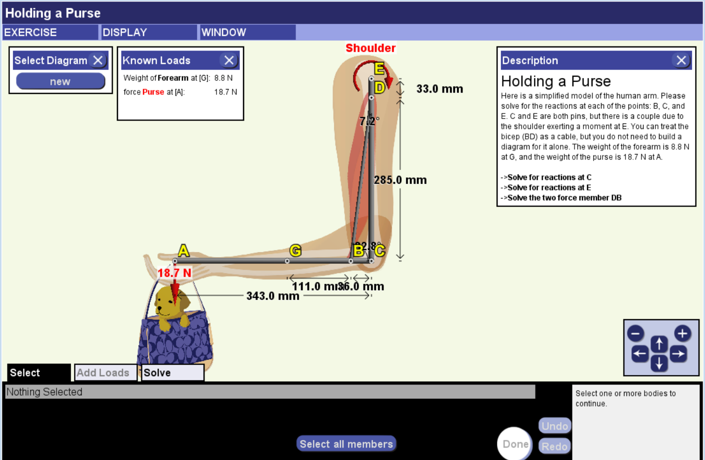
Equilibrium in 2D

Free-body Diagram, Holding a Purse



Equilibrium in 2D

Free-body Diagram, Holding a Purse

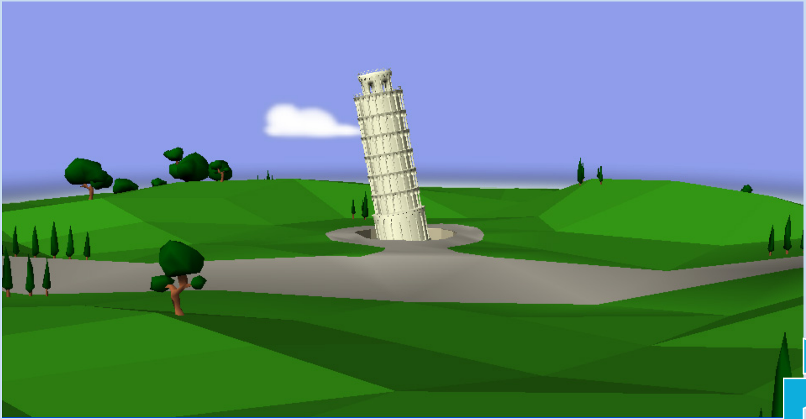


Equilibrium in 2D

Free-body Diagram, Pisa Challenge, A Tower in Masouleh

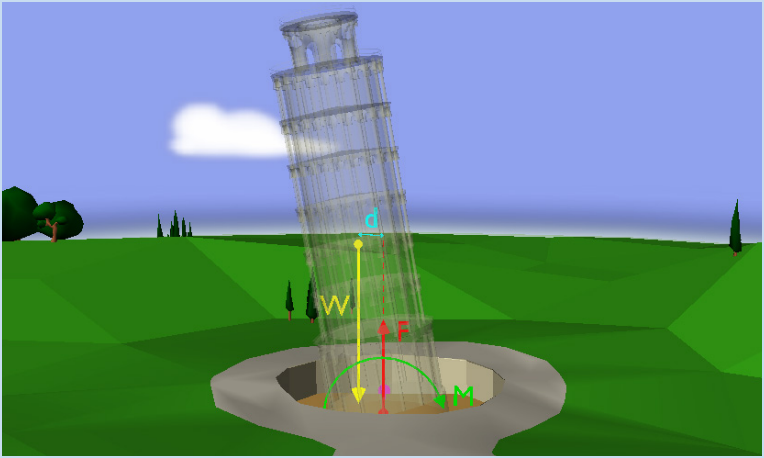
Equilibrium in 2D

Free-body Diagram, Pisa Challenge, A Tower in Masouleh



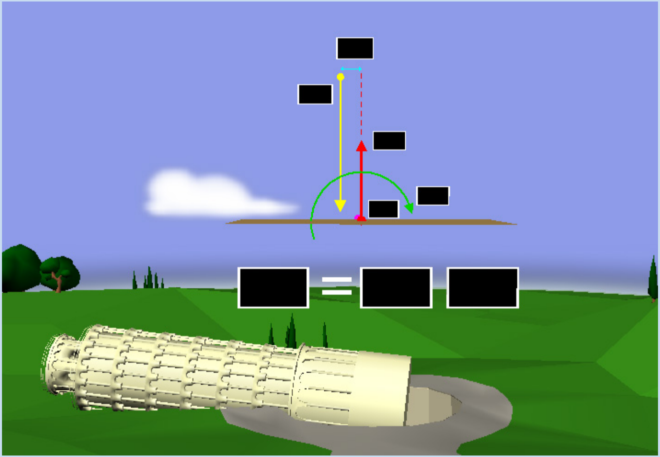
Equilibrium in 2D

Free-body Diagram, Pisa Challenge, A Tower in Masouleh



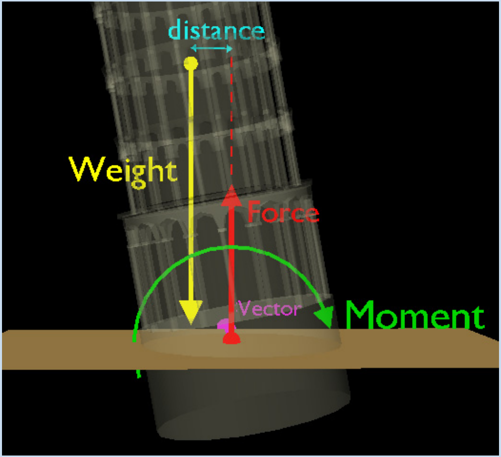
Equilibrium in 2D

Free-body Diagram, Pisa Challenge, A Tower in Masouleh



Equilibrium in 2D

Free-body Diagram, Pisa Challenge, A Tower in Masouleh

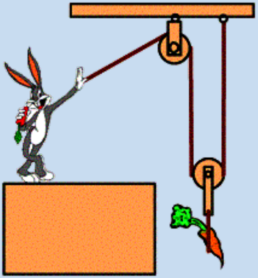


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

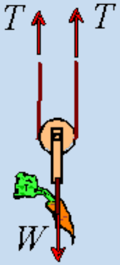


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction



Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

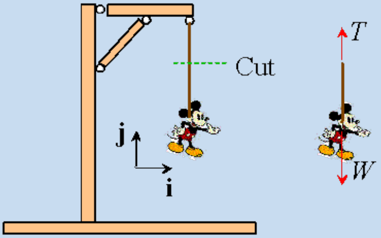
- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

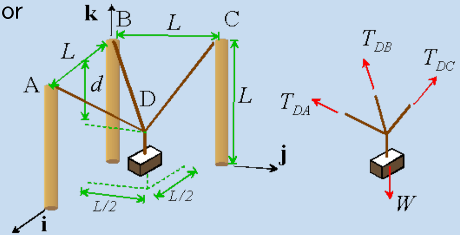


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

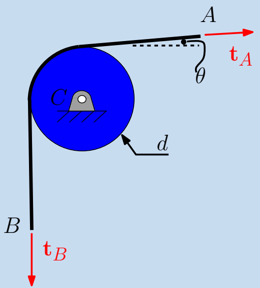


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

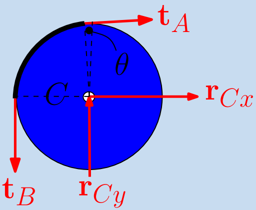


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

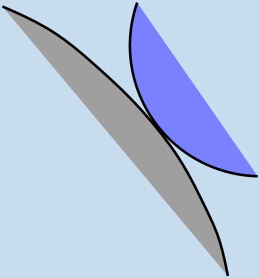


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

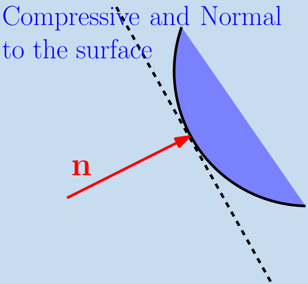


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

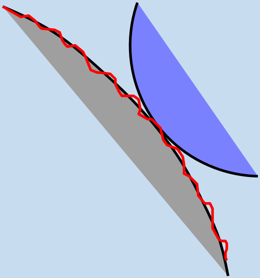


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

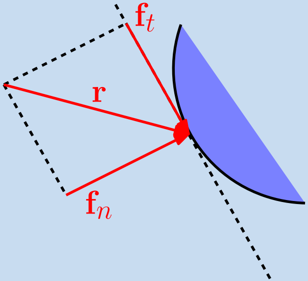


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

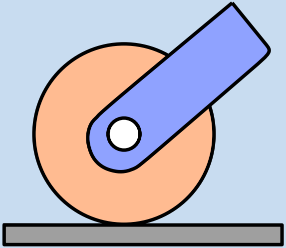


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

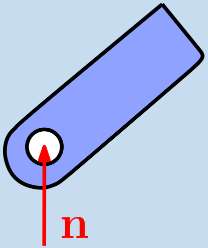


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

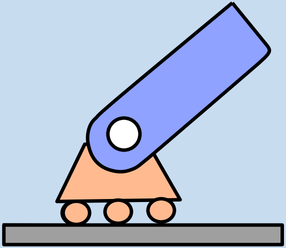


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

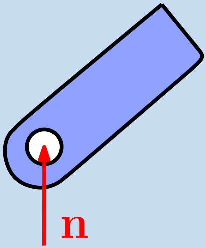


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

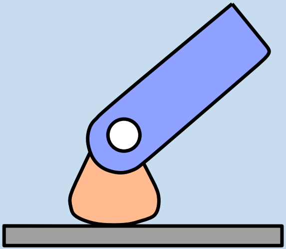


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

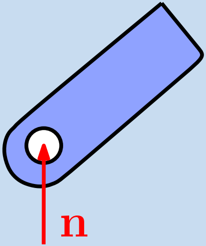


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

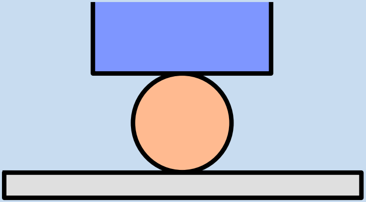


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

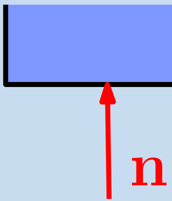


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

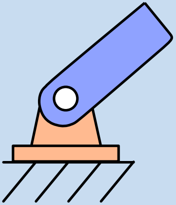


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

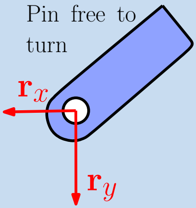


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

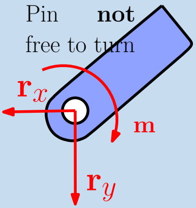


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

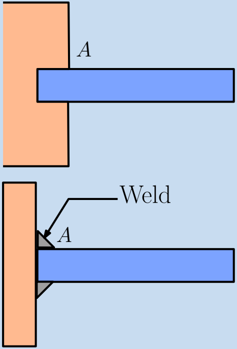


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- **Built-in or fixed connection**
- Gravitation attraction

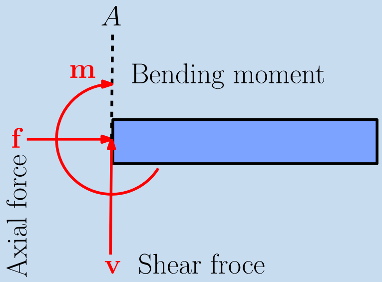


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

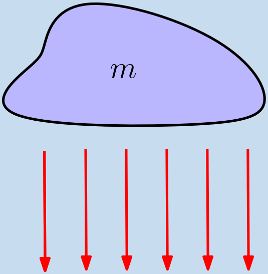


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction

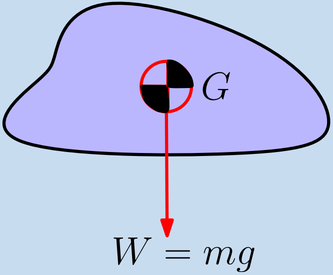


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (Very Important)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction



Equilibrium in 2D

Equilibrium Conditions

- The resultant of all forces and moments acting on a body is zero
- For the moment any point O
- Based on $\sum \mathbf{f} = m\mathbf{a}$, if $\mathbf{v}$ =cet
- Necessary and sufficient conditions for equilibrium
- Statically balanced mechanisms (A part of your exam) It will be the subject of an upcoming course.

$$\sum f_x = 0$$
$$\sum f_y = 0$$
$$\sum m_o = 0$$

Equilibrium in 2D

Equilibrium Conditions

- The resultant of all forces and moments acting on a body is zero
- For the moment any point O
- Based on $\sum \mathbf{f} = m\mathbf{a}$, if $\mathbf{v}$ =cet
- Necessary and sufficient conditions for equilibrium
- Statically balanced mechanisms (A part of your exam) It will be the subject of an upcoming course.

$$\sum f_x = 0$$

$$\sum f_y = 0$$

$$\sum m_o = 0$$

Equilibrium in 2D

Equilibrium Conditions

- The resultant of all forces and moments acting on a body is zero
- For the moment any point O
- Based on $\sum \mathbf{f} = m\mathbf{a}$, if $\mathbf{v}$ =cet
- Necessary and sufficient conditions for equilibrium
- Statically balanced mechanisms (A part of your exam) It will be the subject of an upcoming course.

$$\sum f_x = 0$$
$$\sum f_y = 0$$
$$\sum m_o = 0$$

Equilibrium in 2D

Equilibrium Conditions

- The resultant of all forces and moments acting on a body is zero
- For the moment any point O
- Based on $\sum \mathbf{f} = m\mathbf{a}$, if $\mathbf{v}$ =cet
- Necessary and sufficient conditions for equilibrium
- Statically balanced mechanisms (A part of your exam) It will be the subject of an upcoming course.

$$\sum f_x = 0$$

$$\sum f_y = 0$$

$$\sum m_o = 0$$

Equilibrium in 2D

Equilibrium Conditions

- The resultant of all forces and moments acting on a body is zero
- For the moment any point O
- Based on $\sum \mathbf{f} = m\mathbf{a}$, if $\mathbf{v}$ =cet
- Necessary and sufficient conditions for equilibrium
- Statically balanced mechanisms (A part of your exam) It will be the subject of an upcoming course.

$$\sum f_x = 0$$

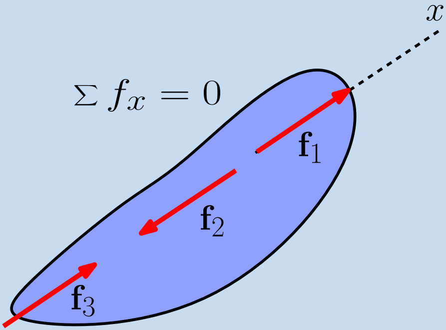
$$\sum f_y = 0$$

$$\sum m_o = 0$$

Equilibrium in 2D

Categories of Equilibrium

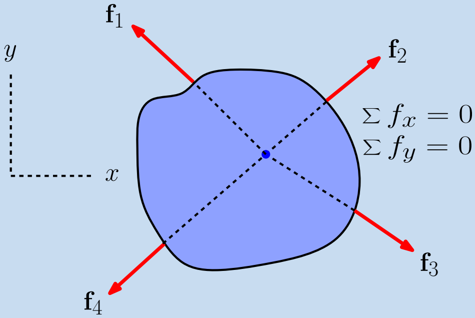
1. Collinear
2. Concurrent at a point
3. Parallel
4. General



Equilibrium in 2D

Categories of Equilibrium

1. Collinear
2. Concurrent at a point
3. Parallel
4. General

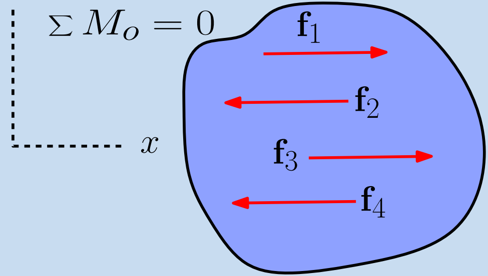


Equilibrium in 2D

Categories of Equilibrium

1. Collinear
2. Concurrent at a point
3. Parallel
4. General

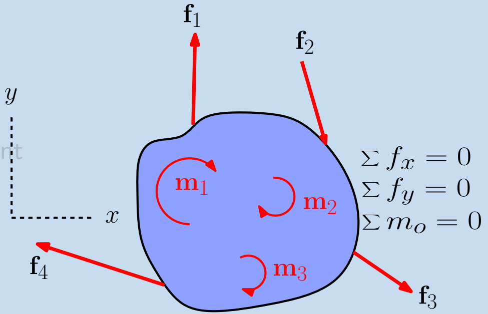
$$\sum f_x = 0$$
$$\sum M_o = 0$$



Equilibrium in 2D

Categories of Equilibrium

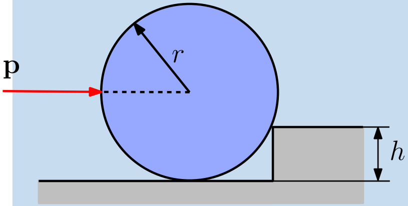
1. Collinear
2. Concurrent at a point
3. Parallel
4. **General**



Equilibrium in 2D

Examples

Determine p required to being rolling?



$$\sin\alpha = \frac{\sqrt{2rh - h^2}}{r}$$

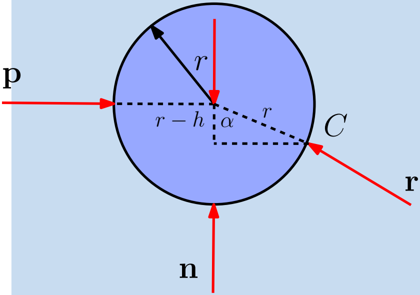
$$\sum m_O = 0$$

$$p(r-h) - mgr\sin\alpha = 0$$

$$p = \frac{mg\sqrt{2rh - h^2}}{r-h}$$

Equilibrium in 2D

Examples



Determine p required to being rolling?

$$\sin\alpha = \frac{\sqrt{2rh-h^2}}{r}$$

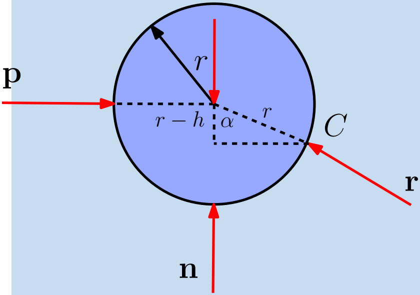
$$\sum m_O = 0$$

$$p(r-h) - mgr\sin\alpha = 0$$

$$p = \frac{mg\sqrt{2rh-h^2}}{r-h}$$

Equilibrium in 2D

Examples



Determine p required to being rolling?

$$\sin\alpha = \frac{\sqrt{2rh - h^2}}{r}$$

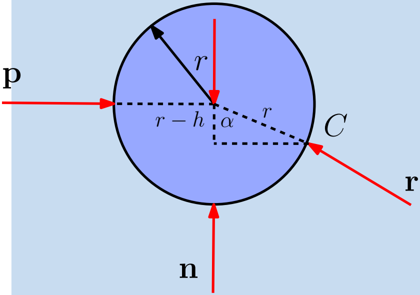
$$\sum m_O = 0$$

$$p(r-h) - mgr\sin\alpha = 0$$

$$p = \frac{mg\sqrt{2rh - h^2}}{r-h}$$

Equilibrium in 2D

Examples



Determine p required to being rolling?

$$\sin\alpha = \frac{\sqrt{2rh - h^2}}{r}$$

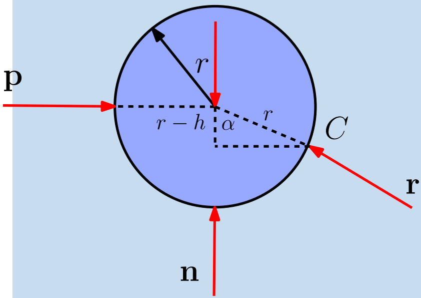
$$\sum m_O = 0$$

$$p(r-h) - mgr\sin\alpha = 0$$

$$p = \frac{mg\sqrt{2rh-h^2}}{r-h}$$

Equilibrium in 2D

Examples



Determine p required to being rolling?

$$\sin\alpha = \frac{\sqrt{2rh - h^2}}{r}$$

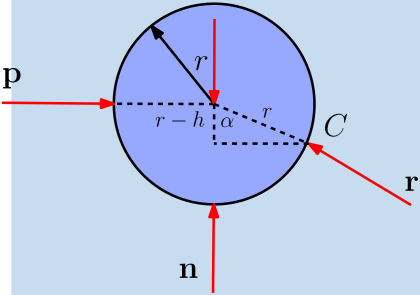
$$\sum m_O = 0$$

$$p(r-h) - mgr\sin\alpha = 0$$

$$p = \frac{mg\sqrt{2rh - h^2}}{r-h}$$

Equilibrium in 2D

Examples



Determine p required to being rolling?

$$\sin\alpha = \frac{\sqrt{2rh - h^2}}{r}$$

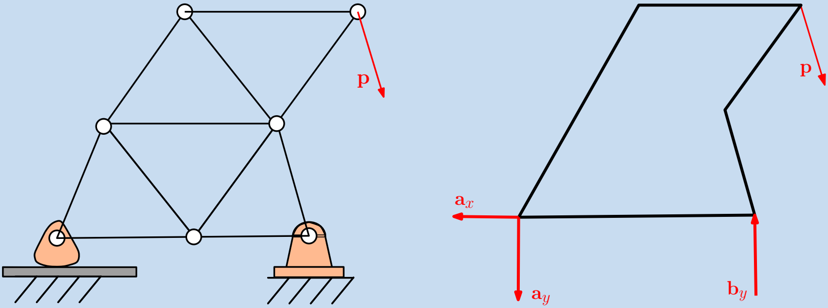
$$\sum m_O = 0$$

$$p(r-h) - mgr\sin\alpha = 0$$

$$p = \frac{mg\sqrt{2rh - h^2}}{r-h}$$

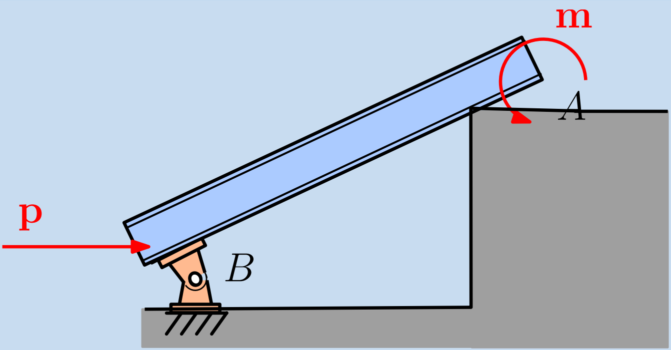
Equilibrium in 2D

Examples of FBD, Plane truss



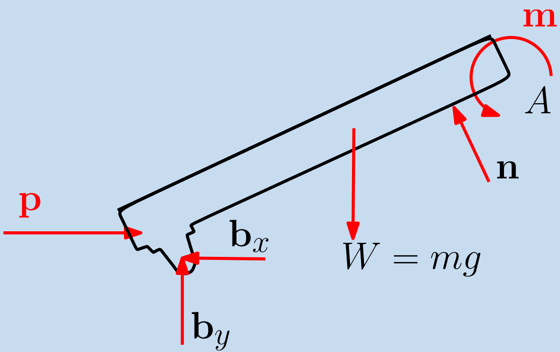
Equilibrium in 2D

Examples of FBD, Beam



Equilibrium in 2D

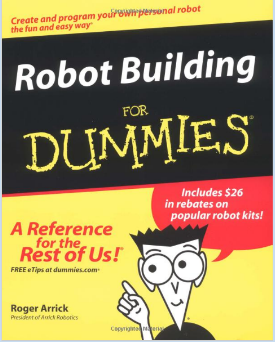
Examples of FBD, Beam



Mechanisms Statically Balanced

The State of the Art

- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the



Mechanisms Statically Balanced

The State of the Art

- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the

Mechanisms Statically Balanced

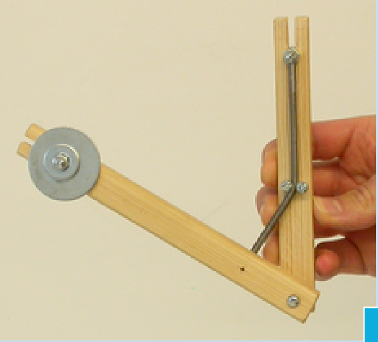
The State of the Art

- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the

Mechanisms Statically Balanced

The State of the Art

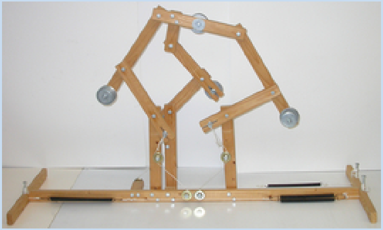
- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the



Mechanisms Statically Balanced

The State of the Art

- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the

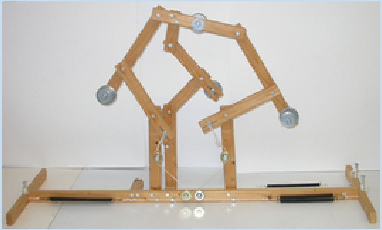


with spring

Mechanisms Statically Balanced

The State of the Art

- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the

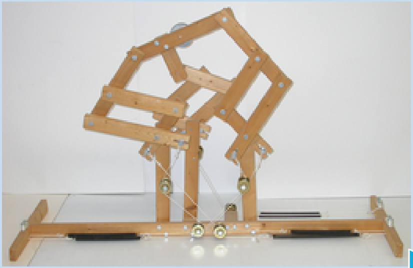


with springs & counterweights

Mechanisms Statically Balanced

The State of the Art

- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the



with springs

Mechanisms Statically Balanced

The State of the Art

- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the

Two Approaches

- From the equilibrium concepts: $\sum \mathbf{f} = 0, \quad \sum \mathbf{m} = 0$
- Using potential energy, U:
  1. Find the expression for $\mathbf{U}$
  2. Its derivative equaled to zero
  3. Solve the system of equations

Mechanisms Statically Balanced

The State of the Art

- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the

Two Approaches

- From the equilibrium concepts:
  $\sum \mathbf{f} = 0, \quad \sum \mathbf{m} = 0$
- Using potential energy, U:
  1. Find the expression for $\mathbf{U}$
  2. Its derivative equaled to zero
  3. Solve the system of equations

Mechanisms Statically Balanced

The State of the Art

- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the

Two Approaches

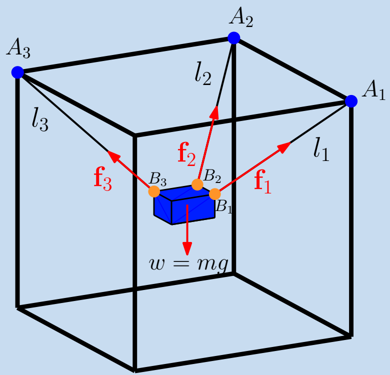
- From the equilibrium concepts:

$$\sum \mathbf{f} = 0, \quad \sum \mathbf{m} = 0$$

- Using potential energy, U:
  1. Find the expression for **U**
  2. Its derivative equaled to zero
  3. Solve the system of equations

3 Suspended Cables

A typical question for exam



- Write the equilibrium equations

$$\sum \mathbf{f} = 0 \qquad \sum \mathbf{m} = 0$$

- Use the concept of two points for representing the vector

$$\sum f_x = \sum f_i \frac{x_{Bi} - x_{Ai}}{l_i} = 0$$

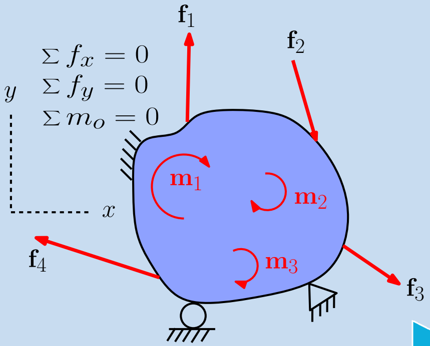
$$\sum f_y = \sum f_i \frac{y_{Bi} - y_{Ai}}{l_i} = 0$$

$$\sum f_z = \sum f_i \frac{z_{Bi} - z_{Ai}}{l_i} = 0$$

Equilibrium in 2D

Constraints and Statical Determinacy

- Necessary and sufficient conditions,
  $\sum \mathbf{f} = 0 \qquad \sum \mathbf{m} = 0$
- n_E: Number of equations, in 2D $n_R = 3$
- n_R: Number of unknowns
- It may happen that
  1. $n_R > n_E$: Statically indeterminate
  2. $n_R = n_E$: Statically determinate
- This refers us to our Adv. Eng. Mathematics

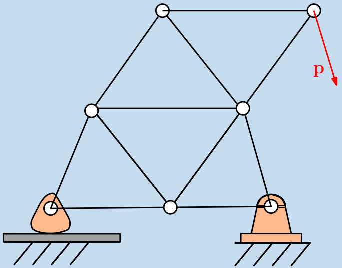


$\mathcal{D}_c = n_R - n_E$ degree of statical indetreminacy

Equilibrium in 2D

Constraints and Statical Determinacy

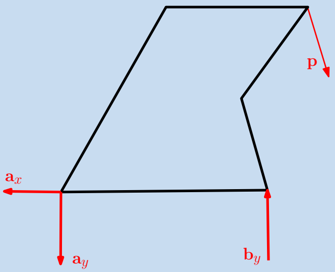
- Necessary and sufficient conditions,
  $\sum \mathbf{f} = 0 \qquad \sum \mathbf{m} = 0$
- n_E: Number of equations, in 2D $n_R = 3$
- n_R: Number of unknowns
- It may happen that
  1. $n_R > n_E$: Statically indeterminate
  2. $n_R = n_E$: Statically determinate
- This refers us to our Adv. Eng. Mathematics



Equilibrium in 2D

Constraints and Statical Determinacy

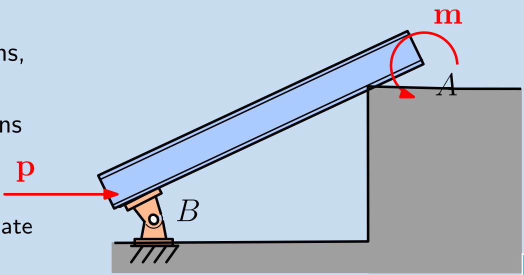
- Necessary and sufficient conditions,
  $\sum \mathbf{f} = 0 \qquad \sum \mathbf{m} = 0$
- n_E: Number of equations, in 2D $n_R = 3$
- n_R: Number of unknowns
- It may happen that
  1. $n_R > n_E$: Statically indeterminate
  2. $n_R = n_E$: Statically determinate
- This refers us to our Adv. Eng. Mathematics



Equilibrium in 2D

Constraints and Statical Determinacy

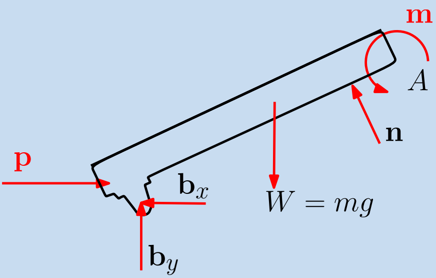
- Necessary and sufficient conditions,
  $\sum \mathbf{f} = 0 \qquad \sum \mathbf{m} = 0$
- n_E: Number of equations, in 2D $n_R = 3$
- n_R: Number of unknowns
- It may happen that
  1. $n_R > n_E$: Statically indeterminate
  2. $n_R = n_E$: Statically determinate
- This refers us to our Adv. Eng. Mathematics



Equilibrium in 2D

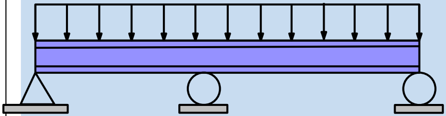
Constraints and Statical Determinacy

- Necessary and sufficient conditions,
  $\sum \mathbf{f} = 0 \qquad \sum \mathbf{m} = 0$
- n_E: Number of equations, in 2D $n_R = 3$
- n_R: Number of unknowns
- It may happen that
  1. $n_R > n_E$: Statically indeterminate
  2. $n_R = n_E$: Statically determinate
- This refers us to our Adv. Eng. Mathematics



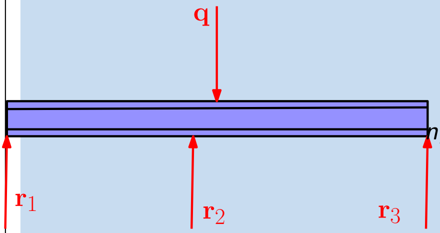
Equilibrium in 2D (A First Step Toward 3D)

Examples of Statically Indeterminate Structures



Equilibrium in 2D (A First Step Toward 3D)

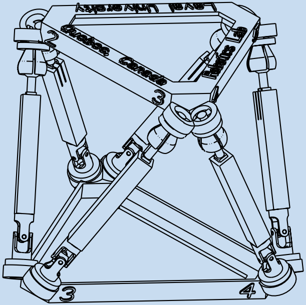
Examples of Statically Indeterminate Structures



$n_R > n_E$ $\mathcal{D}_c = n_R - n_E = 1$

Equilibrium in 2D (A First Step Toward 3D)

Examples of Statically Indeterminate Structures



6-UPS

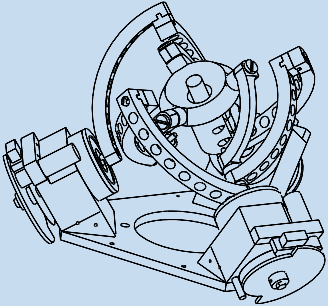
Your exam questions

$n_R = ?$ and $n_E = ?$

- Constraints for a R, U, C and S joints
- Number of unknowns per limb
- Number of equations per limb
- Number of equations for the platform
- Repeat the same for a 6-SPS
- Repeat the same for Agile Eye

Equilibrium in 2D (A First Step Toward 3D)

Examples of Statically Indeterminate Structures



Agile Eye

Your exam questions

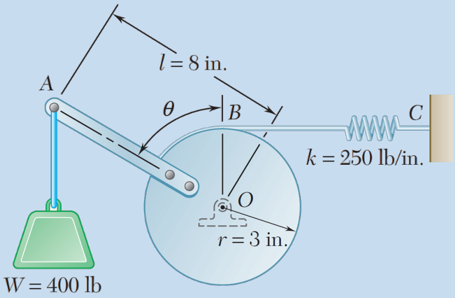
n_R =? and n_E =?

- Constraints for a R, U, C and S joints
- Number of unknowns per limb
- Number of equations per limb
- Number of equations for the platform
- Repeat the same for a 6-S<u>P</u>S
- Repeat the same for Agile Eye

Equilibrium in 2D

Examples with spring

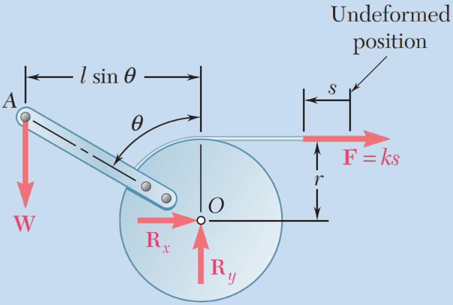
Determine the position of equilibrium



- Free-body diagram
- $s = r\theta$
- $\sum m_O = 0$
- $Wl \sin\theta - r(kr\theta) = 0$
- $\sin\theta = \dfrac{kr^2}{Wl}\theta$
- Propose some methods to solve it!
- $\sin\theta = \theta - \dfrac{\theta^3}{3!} + \dfrac{\theta^5}{4!} - \ldots$

Equilibrium in 2D

Examples with spring



Determine the position of equilibrium

- Free-body diagram
- $s = r\theta$
- $\sum m_O = 0$
- $Wl \sin\theta - r(kr\theta) = 0$
- $\sin\theta = \dfrac{kr^2}{Wl}\theta$
- Propose some methods to solve it!
- $\sin\theta = \theta - \dfrac{\theta^3}{3!} + \dfrac{\theta^5}{4!} - \dots$

Equilibrium in 3D

Examples with spring

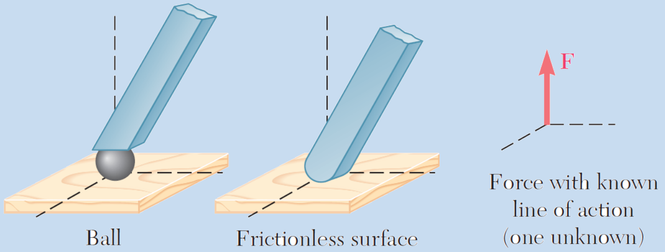
- Extension of 2D and nothing else
- Most of the concepts are left to you

$$\sum \mathbf{f} = \mathbf{0} \quad \text{or} \quad \begin{cases} \sum f_x = 0 \\ \sum f_y = 0 \\ \sum f_z = 0 \end{cases}$$

$$\sum \mathbf{m} = \mathbf{0} \quad \text{or} \quad \begin{cases} \sum m_x = 0 \\ \sum m_y = 0 \\ \sum m_z = 0 \end{cases}$$

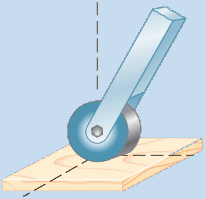
Equilibrium in 3D

Reactions at supports and connections

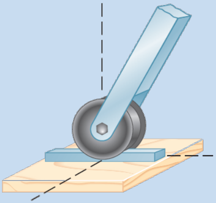


Equilibrium in 3D

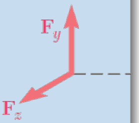
Reactions at supports and connections



Roller on rough surface



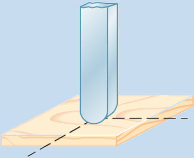
Wheel on rail



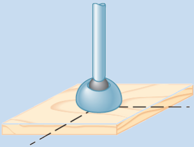
Two force components

Equilibrium in 3D

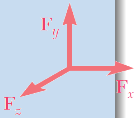
Reactions at supports and connections



Rough surface



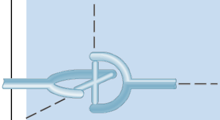
Ball and socket



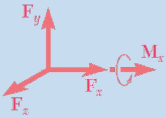
Three force components

Equilibrium in 3D

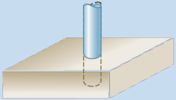
Reactions at supports and connections



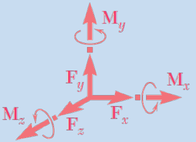
Universal joint



Three force components and one couple



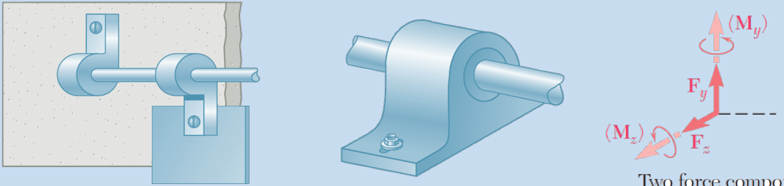
Fixed support



Three force components and three couples

Equilibrium in 3D

Reactions at supports and connections

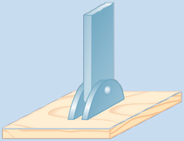


Hinge and bearing supporting radial load only

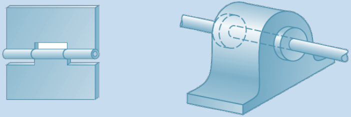
Two force components (and two couples)

Equilibrium in 3D

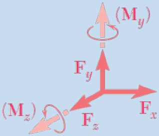
Reactions at supports and connections



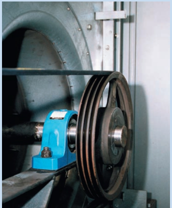
Pin and bracket



Hinge and bearing supporting axial thrust and radial load

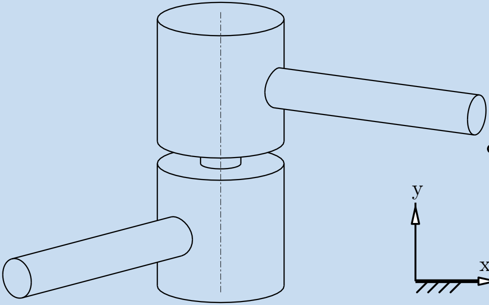


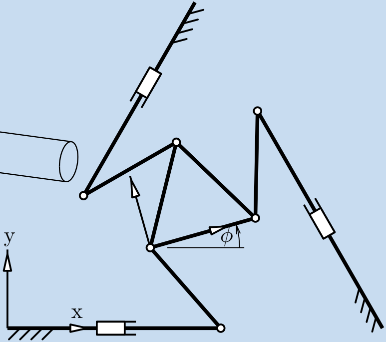
Three force components (and two couples)



Equilibrium in 3D

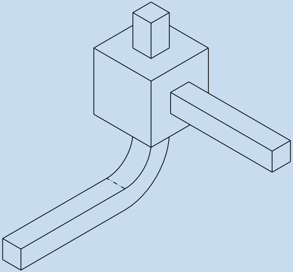
Reactions at supports and connections- Exam questions

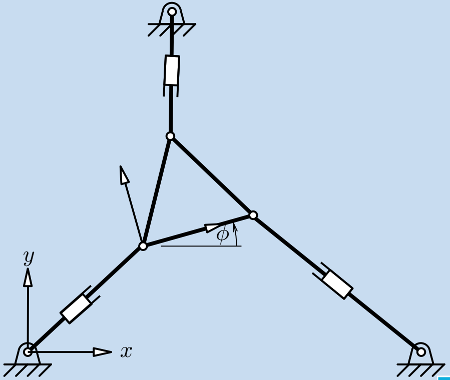




Equilibrium in 3D

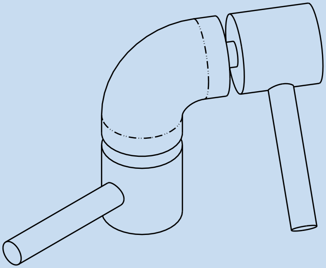
Reactions at supports and connections- Exam questions

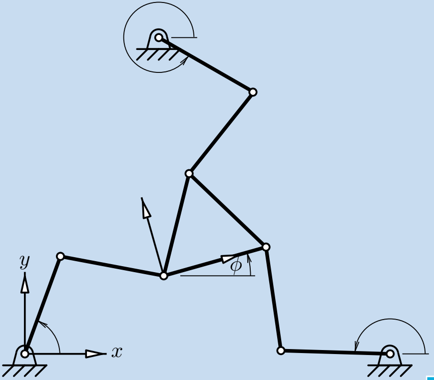




Equilibrium in 3D

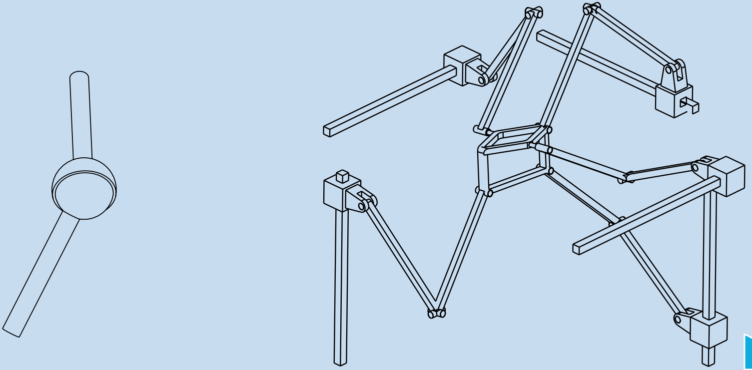
Reactions at supports and connections- Exam questions





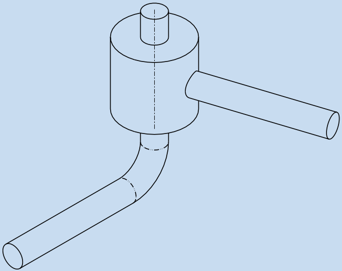
Equilibrium in 3D

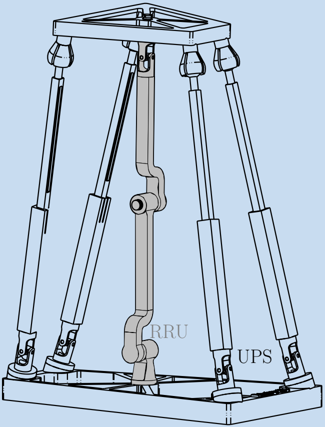
Reactions at supports and connections- Exam questions



Equilibrium in 3D

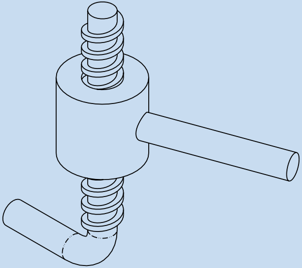
Reactions at supports and connections- Exam questions

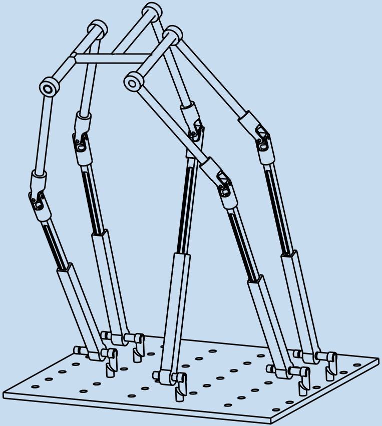




Equilibrium in 3D

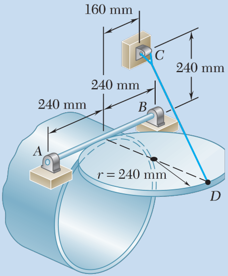
Reactions at supports and connections- Exam questions





Equilibrium in 3D

Example



$$\mathbf{W} = -mg\mathbf{j} = (-294\text{N})\mathbf{j}$$

$$\overrightarrow{DC} = -(480\text{mm})\mathbf{i} + (240\text{mm})\mathbf{j} - (160\text{mm})\mathbf{k}$$

$$\mathbf{T} = T\frac{\overrightarrow{DC}}{DC} = -\frac{6}{7}T\mathbf{i} + \frac{3}{7}T\mathbf{j} - \frac{2}{7}T\mathbf{k}$$

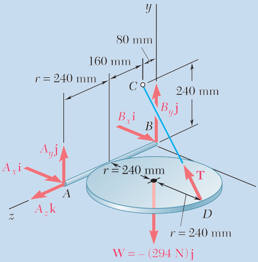
$$\sum \mathbf{f} = 0$$

$$A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} + B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k} + \mathbf{T} - (294\text{N})\mathbf{j} = \mathbf{0}$$

For the moment!

Equilibrium in 3D

Example



$$\mathbf{W} = -mg\mathbf{j} = (-294\text{N})\mathbf{j}$$

$$\overrightarrow{DC} = -(480\text{mm})\mathbf{i} + (240\text{mm})\mathbf{j} - (160\text{mm})\mathbf{k}$$

$$\mathbf{T} = T\frac{\overrightarrow{DC}}{DC} = -\frac{6}{7}T\mathbf{i} + \frac{3}{7}T\mathbf{j} - \frac{2}{7}T\mathbf{k}$$

$$\sum \mathbf{f} = 0$$

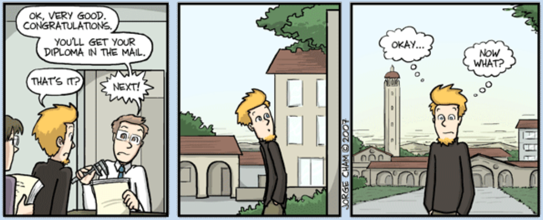
$$A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} + B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k} + \mathbf{T} - (294\text{N})\mathbf{j} = \mathbf{0}$$

For the moment!

Some Hints for the Exam

Prepare yourself, All the sample problems and

- Section A: Equilibrium in two dimensions:
  3/5, 3/10, 3/31 3/46, 3/55
- Section B: Equilibrium in three dimensions:
  3/63, 3/73, 3/79, 3/81, 3/85
- Chapter Review: 3/98, 3/101, 3/107, 3/108, 3/109
- FBD, Equilibrium conditions, Question exams (in slides)



THIS MARKS THE END OF THE THIRD CHAPTER IN THE PHD SAGA..!