

Static and Strength of Materials-

Chapter 3-Equilibrium

Mehdi Tale Masouleh



October 17, 2013



Introductions

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- A new definition: Statics deals with the description of the force conditions necessary and sufficient to maintain the equilibrium of engineering structures.
- Form your secondary:

$$\mathbf{r} = \sum \mathbf{f}_i = 0 \quad \mathbf{m} = \sum \mathbf{m}_i = 0$$

- Both are necessary and sufficient
- All physical bodies are 3D
- In some cases we can treat them as 2D
- When forces lie on single plane
- Can be projected onto a single plane

The same as Chapter 2:

- Equilibrium in 2D
- Equilibrium in 3D



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The same as Chapter 2:

- 1 Equilibrium in 2D
- 2 Equilibrium in 3D



Equilibrium in 2D

Definition of Mechanical System

- By J. Meriam: A body or group of bodies which can be conceptually isolated from all other bodies
- By Concise Oxford Dictionary: Complex whole, set of connected things or parts, organized body of material or immaterial things
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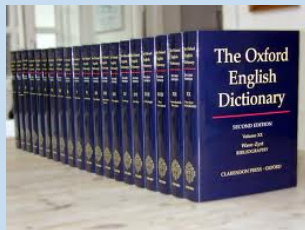




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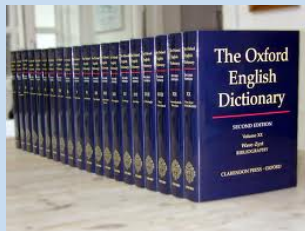




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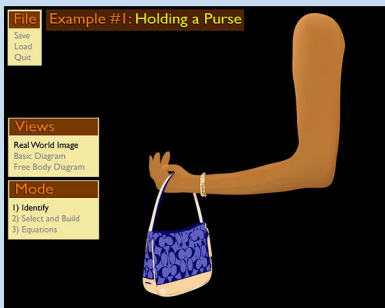




Equilibrium in 2D

Free-body Diagram

- The free-body diagram is the most important single step in the solution of problems in mechanics.
- a diagrammatic representation of the isolated system treated as single body
- Only after such a diagram has been carefully drawn should the equilibrium equations be written





Equilibrium in 2D

Free-body Diagram, Holding a Purse

File Example #1: Holding a Purse

- Save
- Load
- Quit

Views

- Real World Image
- Basic Diagram
- Free Body Diagram

Mode

- 1) Identify
- 2) Select and Build
- 3) Equations

The central image shows a hand holding a blue patterned purse. The hand is brown and has a gold bracelet. The background is black.



Equilibrium in 2D

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Example #1: Holding a Purse

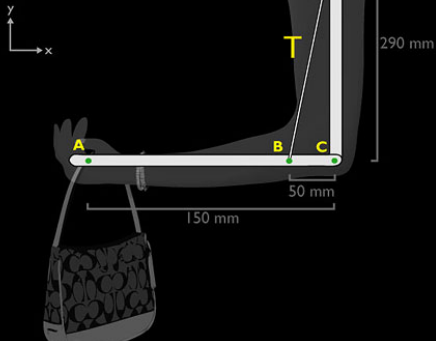
The woman's purse weighs 2 kilograms and her forearm weighs 9 Newtons.
Solve for tension at \overline{BD} .

Views

- Real World Image
- Basic Diagram
- Free Body Diagram
- Degrees
- Forces
- Distances

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FBD Tools

Free Body Diagram #1
Free Body Diagram #2

Forces

- Force
- Weight

Connectors

- pin
- fixed

Structures

- beam
- cable

Points

- point
- G



Equilibrium in 2D

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Mode

- 1) Identify
- 2) Select and Build
- 3) Equations

Check!

FBD Tools

Free Body Diagram #1
Free Body Diagram #2

Forces

- Force
- Weight

Connectors

- pin
- fixed

Structures

- beam
- cable

Points

- point



Equilibrium in 2D

Free-body Diagram, Holding a Purse

File Example #1: Holding a Purse

Save
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Quit

The woman's purse weighs 3 Newtons and her forearm weighs 9 Newtons.
Solve for tension at B.

Views

- Real World Image
- Basic Diagram
- Free Body Diagram
- Degrees
- Forces
- Distances

Mode

- 1) Identify
- 2) Select and Build
- 3) Equations

Equations

$\sum F_y = F_A + F_{AC} + T_v + C_v = 0$

Check!



Equilibrium in 2D

Free-body Diagram, Holding a Purse

Holding a Purse

EXERCISE DISPLAY WINDOW

Select Diagram

Known Loads

Weight of Forearm at [G]: 8.8 N
force Purse at [A]: 18.7 N

Shoulder

33.0 mm

7.2°

285.0 mm

18.7 N

343.0 mm

111.0 mm

86.0 mm

Description

Holding a Purse

Here is a simplified model of the human arm. Please solve for the reactions at each of the points: B, C, and E. C and E are both pins, but there is a couple due to the bicep (BD) as a cable, but you do not need to build a diagram for it alone. The weight of the forearm is 8.8 N at G, and the weight of the purse is 18.7 N at A.

->Solve for reactions at C
->Solve for reactions at E
->Solve the two force member DB

Select Add Loads Solve

Nothing Selected

Select one or more bodies to continue.



Equilibrium in 2D

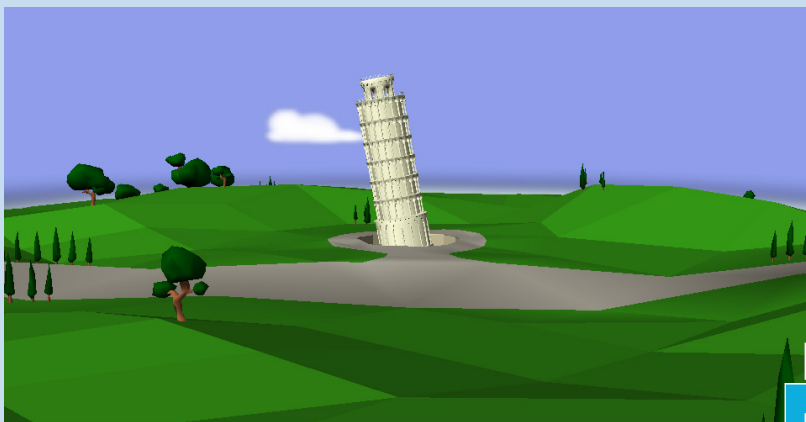
Free-body Diagram, Pisa Challenge, A Tower in Masouleh





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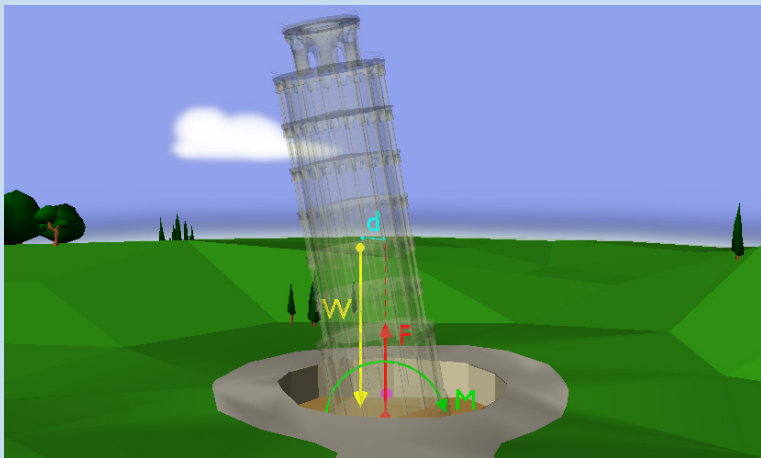
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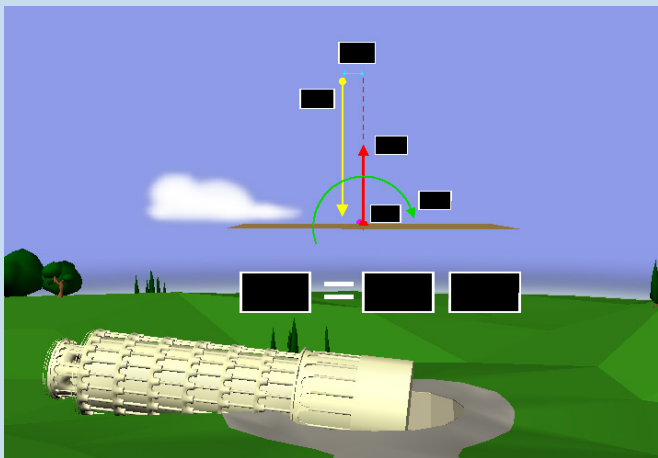
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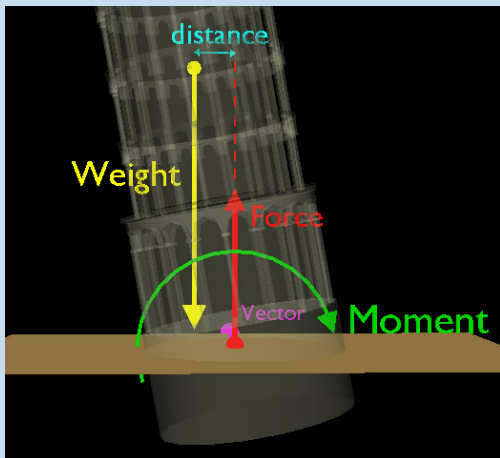
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Equilibrium in 2D

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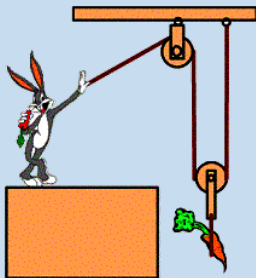


Equilibrium in 2D

Free-body Diagram, Modeling the Action Forces.

Figure 3/1 (**Very Important**)

- Flexible cable, belt, chain, or rope
- Smooth surfaces
- Rough Surfaces
- Roller support
- Pin connection
- Built-in or fixed connection
- Gravitation attraction



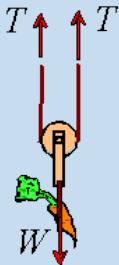


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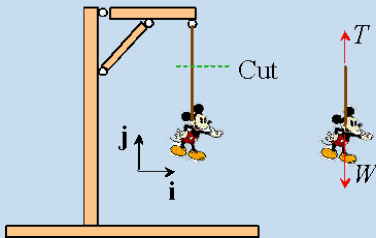


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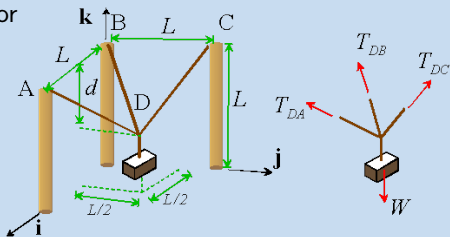


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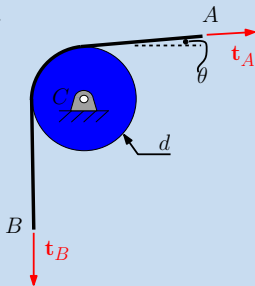


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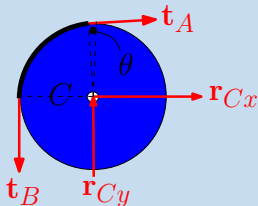


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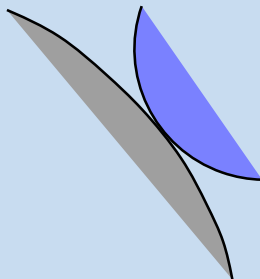


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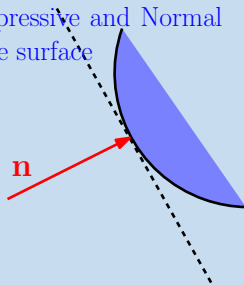
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Compressive and Normal
to the surface



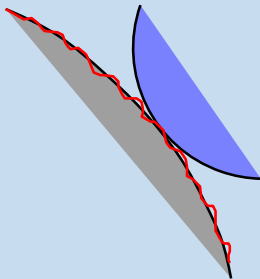


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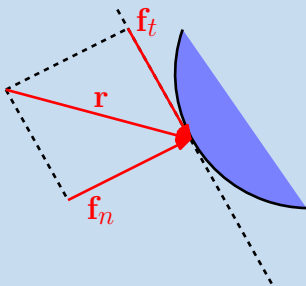


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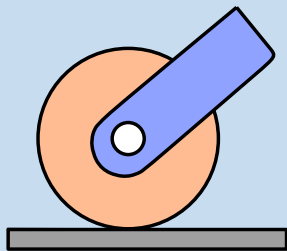


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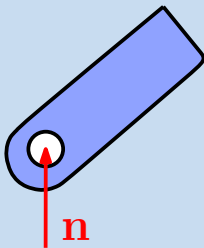


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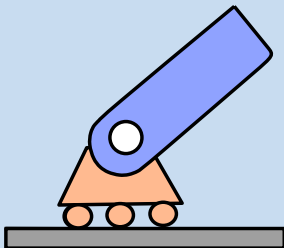


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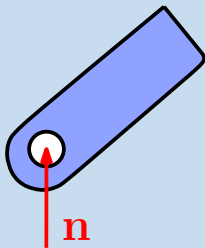


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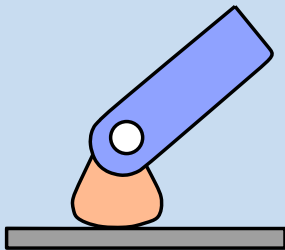


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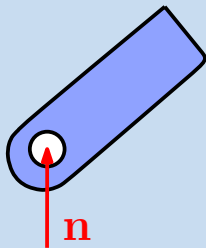


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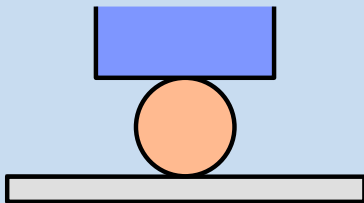


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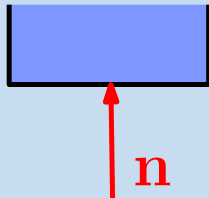


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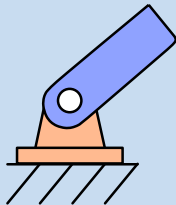


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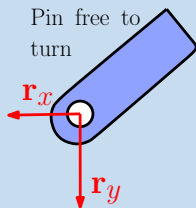


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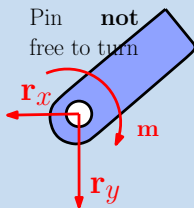


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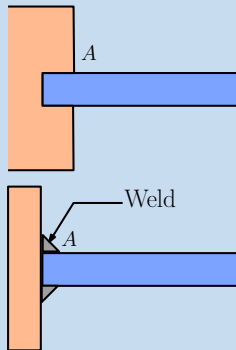


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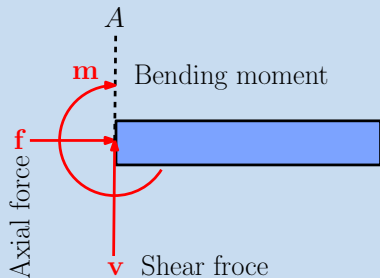


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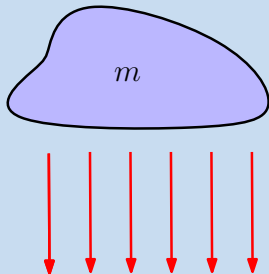


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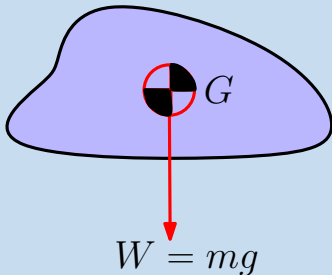


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Equilibrium in 2D

Equilibrium Conditions

- The resultant of all forces and moments acting on a body is zero
- For the moment any point O
- Based on $\sum \mathbf{f} = m\mathbf{a}$, if $\mathbf{v} = \text{cet}$
- Necessary and sufficient conditions for equilibrium
- Statically balanced mechanisms
(A part of your exam)
It will be the subject of an upcoming course.

$$\sum f_x = 0$$

$$\sum f_y = 0$$

$$\sum m_o = 0$$



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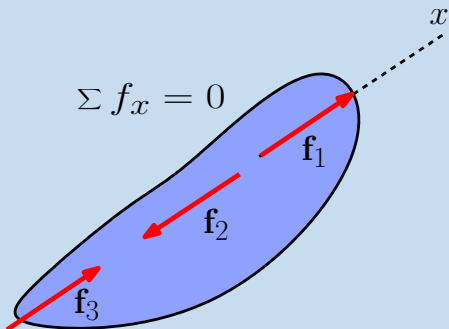
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Equilibrium in 2D

Categories of Equilibrium

- 1 Collinear
- 2 Concurrent at a point
- 3 Parallel
- 4 General

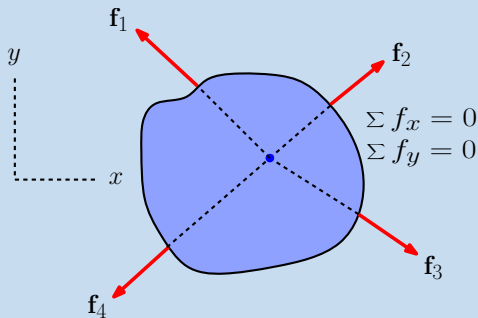




Equilibrium in 2D

Categories of Equilibrium

- 1 Collinear
- 2 **Concurrent at a point**
- 3 Parallel
- 4 General





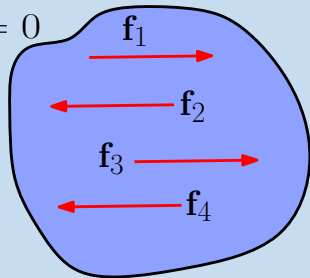
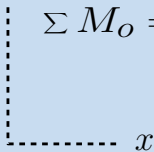
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$$y \quad \sum f_x = 0$$

$$\sum M_O = 0$$

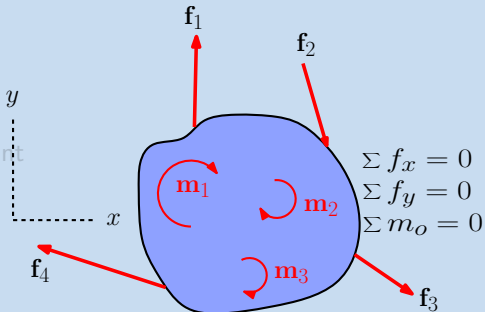




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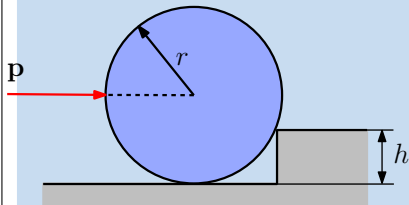




Equilibrium in 2D

Examples

Determine p required to be rolling?



$$\sin \alpha = \frac{\sqrt{2rh - h^2}}{r}$$

$$\sum m_O = 0$$

$$p(r - h) - mgr \sin \alpha = 0$$

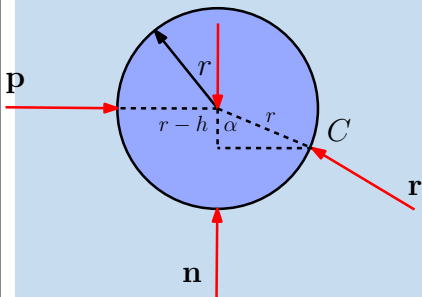
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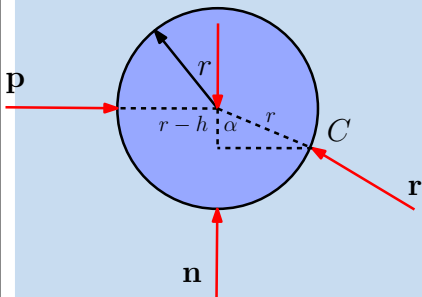
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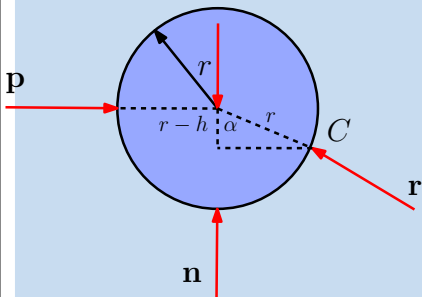
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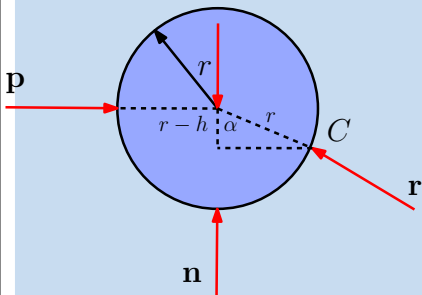
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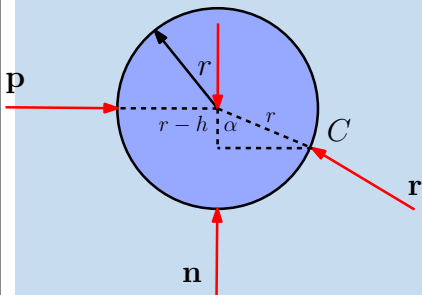
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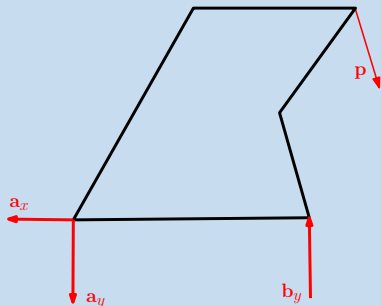
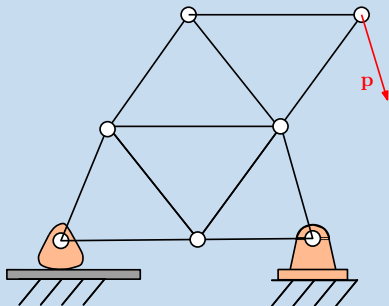
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Equilibrium in 2D

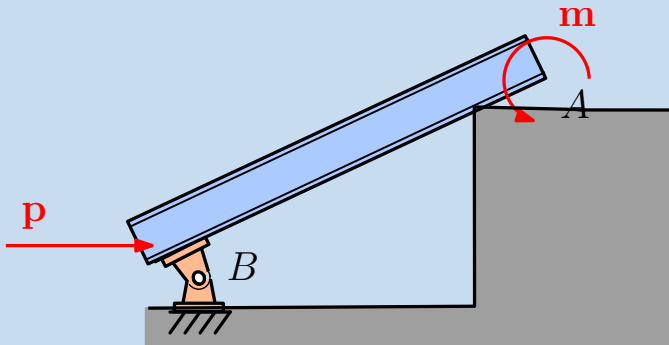
Examples of FBD, Plane truss





Equilibrium in 2D

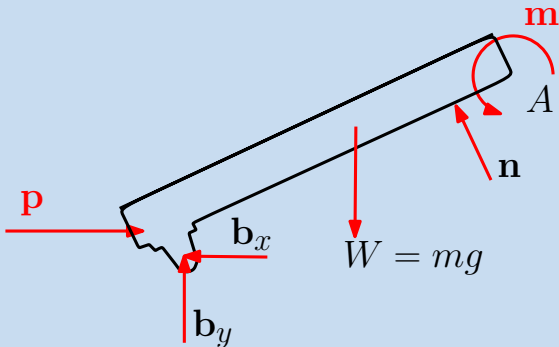
Examples of FBD, Beam





Equilibrium in 2D

Examples of FBD, Beam

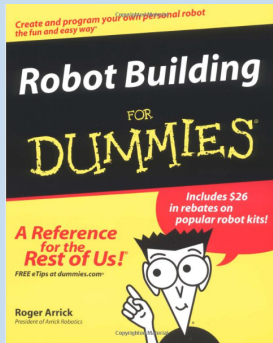




Mechanisms Statically Balanced

The State of the Art

- A major step when building a robot
- Involves ensuring that the motors do not contribute towards supporting the mechanism's weight, for any of the possible configurations, *without the help of motors and brakes*
- This result can be obtained by using counterweights or springs
- How to formulate the





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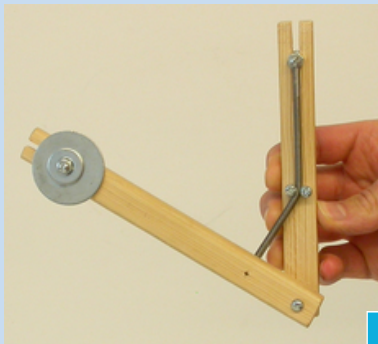




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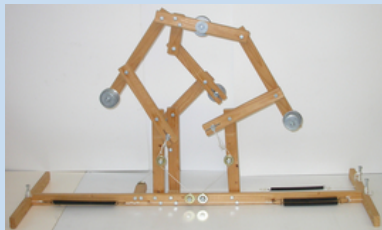
with spring



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with springs & counterweights



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Two Approaches

- From the equilibrium concepts:

$$\sum \mathbf{f} = 0, \quad \sum \mathbf{m} = 0$$

- Using potential energy, U :

- 1 Find the expression for U
- 2 Its derivative equaled to zero
- 3 Solve the system of equations



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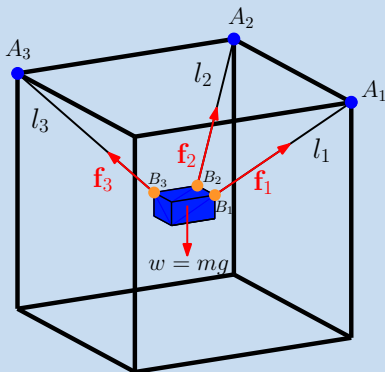
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3 Suspended Cables

A typical question for exam



- Write the equilibrium equations

$$\sum \mathbf{f} = 0 \quad \sum \mathbf{m} = 0$$

- Use the concept of two points for representing the vector

$$\sum f_x = \sum f_i \frac{x_{Bi} - x_{Ai}}{l_i} = 0$$

$$\sum f_y = \sum f_i \frac{y_{Bi} - y_{Ai}}{l_i} = 0$$

$$\sum f_z = \sum f_i \frac{z_{Bi} - z_{Ai}}{l_i} = 0$$

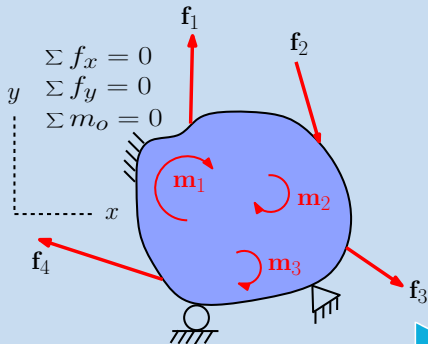




Equilibrium in 2D

Constraints and Static Determinacy

- Necessary and sufficient conditions,
 $\sum \mathbf{f} = 0 \quad \sum \mathbf{m} = 0$
- n_E : Number of equations, in 2D $n_R = 3$
- n_R : Number of unknowns
- It may happen that
 - $n_R > n_E$:
Statically indeterminate
 - $n_R = n_E$:
Statically determinate
- This refers us to our Adv. Eng. Mathematics



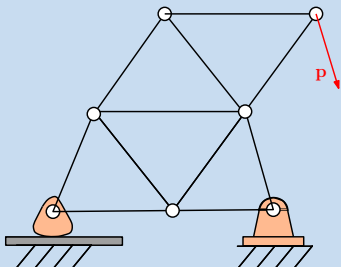
$D_c = n_R - n_E$ degree of statical indetremancy



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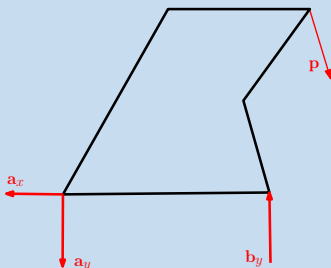




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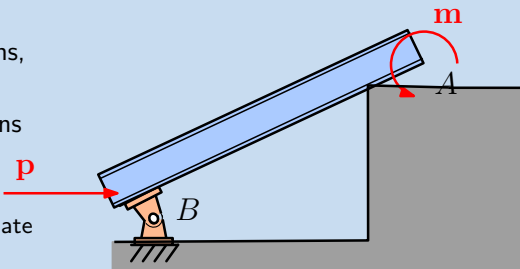




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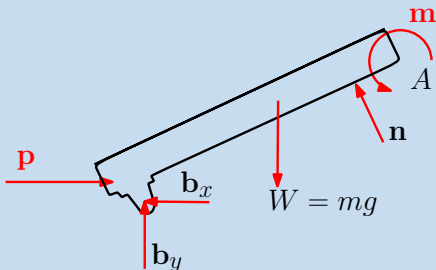




Equilibrium in 2D

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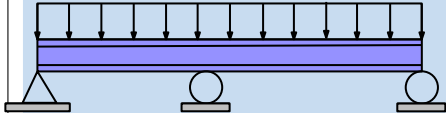
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Equilibrium in 2D (A First Step Toward 3D)

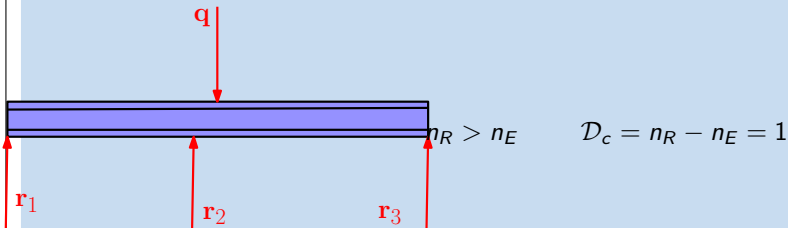
Examples of Statically Indeterminate Structures





Equilibrium in 2D (A First Step Toward 3D)

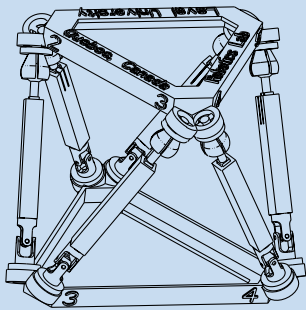
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Equilibrium in 2D (A First Step Toward 3D)

Examples of Statically Indeterminate Structures



6-UPS

Your exam questions

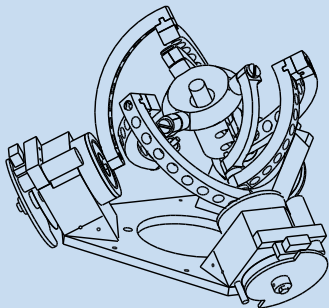
$$n_R = ? \text{ and } n_E = ?$$

- Constraints for a R, U, C and S joints
- Number of unknowns per limb
- Number of equations per limb
- Number of equations for the platform
- Repeat the same for a 6-SPS
- Repeat the same for Agile Eye



Equilibrium in 2D (A First Step Toward 3D)

Examples of Statically Indeterminate Structures



Agile Eye

Your exam questions

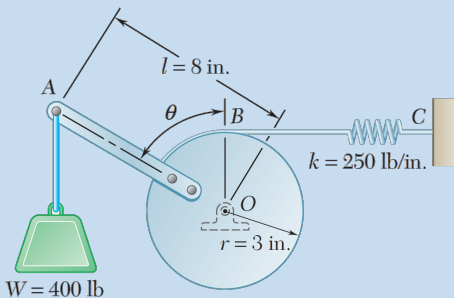
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Equilibrium in 2D

Examples with spring



Determine the position of equilibrium

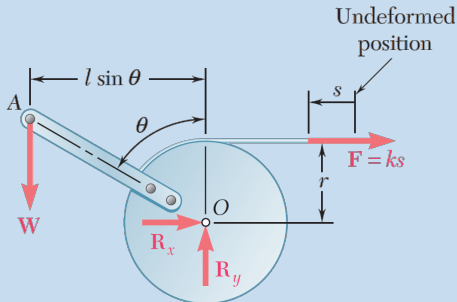
- Free-body diagram
- $s = r\theta$
- $\sum m_O = 0$
- $Wl \sin \theta - r(kr\theta) = 0$
- $\sin \theta = \frac{kr^2}{Wl}$
- Propose some methods to solve it!
- $\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{4!} - \dots$



Equilibrium in 2D

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Equilibrium in 3D

Examples with spring

- Extension of 2D and nothing else
- Most of the concepts are left to you

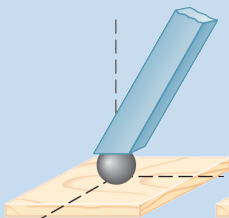
$$\sum \mathbf{f} = \mathbf{0} \quad \text{or} \quad \begin{cases} \sum f_x = 0 \\ \sum f_y = 0 \\ \sum f_z = 0 \end{cases}$$

$$\sum \mathbf{m} = \mathbf{0} \quad \text{or} \quad \begin{cases} \sum m_x = 0 \\ \sum m_y = 0 \\ \sum m_z = 0 \end{cases}$$

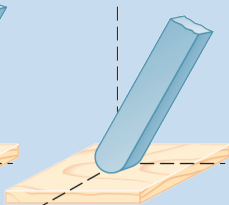


Equilibrium in 3D

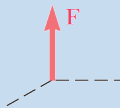
Reactions at supports and connections



Ball



Frictionless surface

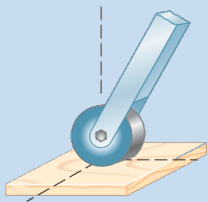


Force with known
line of action
(one unknown)

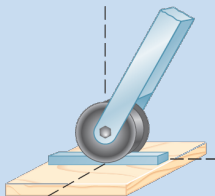


Equilibrium in 3D

Reactions at supports and connections



Roller on rough surface



Wheel on rail

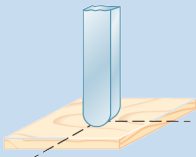


Two force components

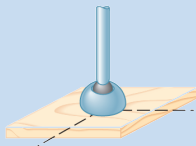


Equilibrium in 3D

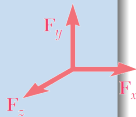
Reactions at supports and connections



Rough surface



Ball and socket

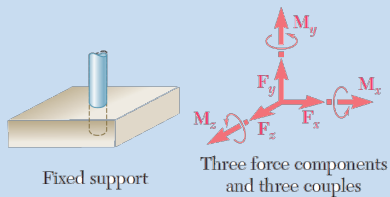
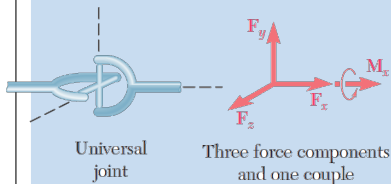


Three force components



Equilibrium in 3D

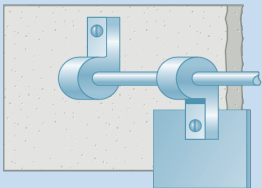
Reactions at supports and connections



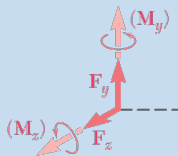
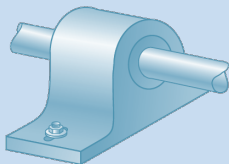


Equilibrium in 3D

Reactions at supports and connections



Hinge and bearing supporting radial load only

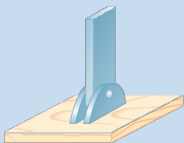


Two force components
(and two couples)

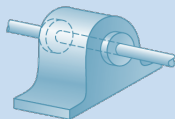


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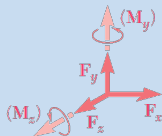
Reactions at supports and connections



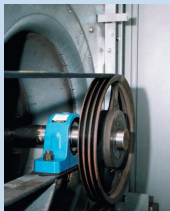
Pin and bracket



Hinge and bearing supporting
axial thrust and radial load



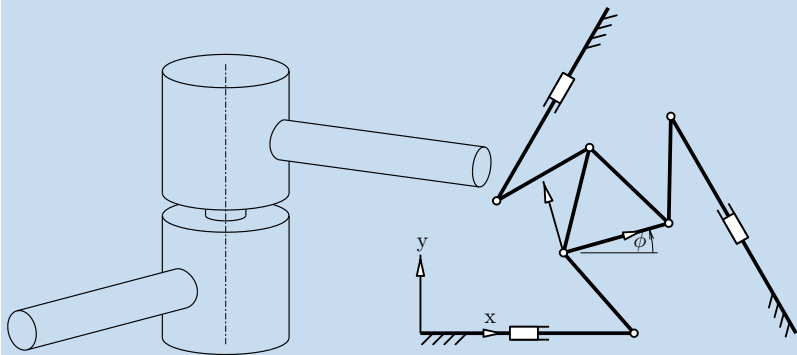
Three force components
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Equilibrium in 3D

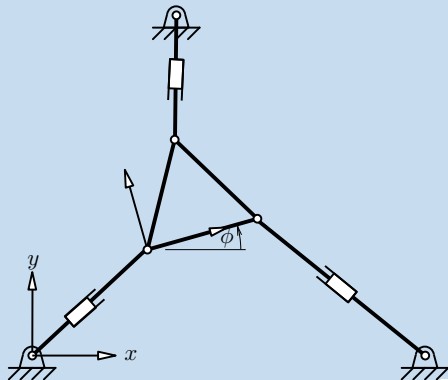
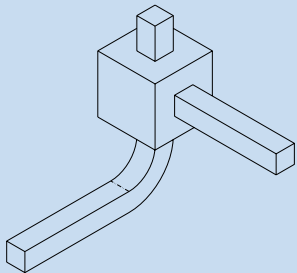
Reactions at supports and connections- Exam questions





Equilibrium in 3D

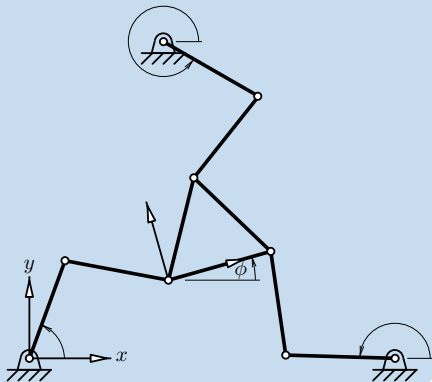
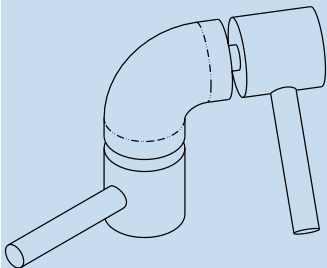
Reactions at supports and connections- Exam questions





Equilibrium in 3D

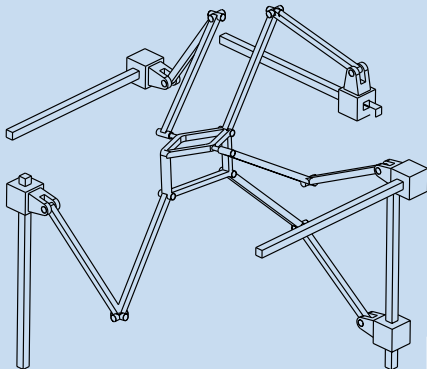
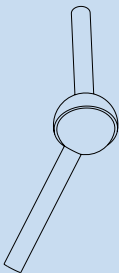
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Equilibrium in 3D

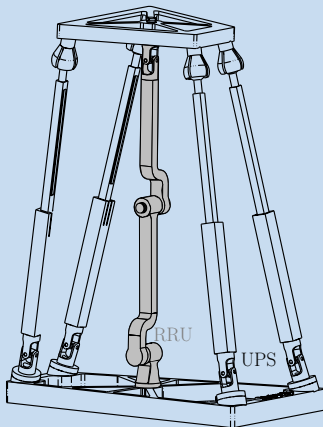
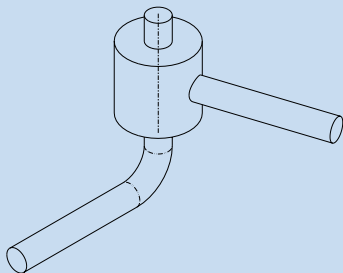
Reactions at supports and connections- Exam questions





Equilibrium in 3D

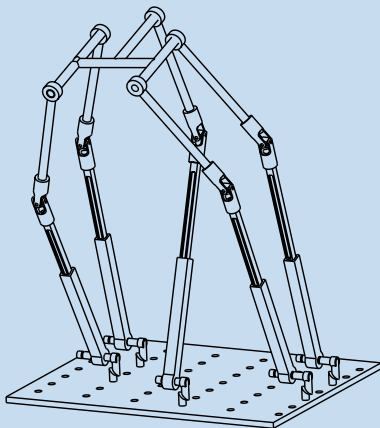
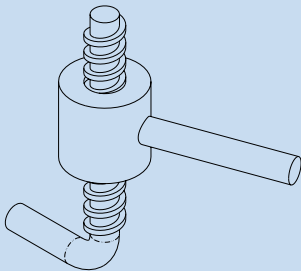
Reactions at supports and connections- Exam questions





Equilibrium in 3D

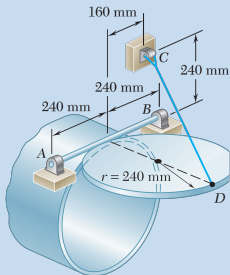
Reactions at supports and connections- Exam questions





Equilibrium in 3D

Example



$$W = -mgj = (-294\text{N})j$$

$$\vec{DC} = -(480\text{mm})i + (240\text{mm})j - (160\text{mm})k$$

$$T = T \frac{\vec{DC}}{DC} = -\frac{6}{7}Ti + \frac{3}{7}Tj - \frac{2}{7}Tk$$

$$\sum \mathbf{f} = 0$$

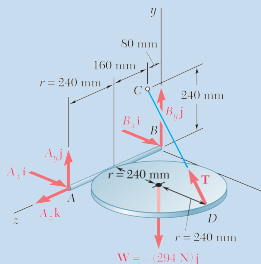
$$A_x i + A_y j + A_z k + B_x i + B_y j + B_z k + T - (294\text{N})j = 0$$

For the moment!



Equilibrium in 3D

Example



$$\mathbf{W} = -mg\mathbf{j} = (-294\text{N})\mathbf{j}$$

$$\overrightarrow{DC} = -(480\text{mm})\mathbf{i} + (240\text{mm})\mathbf{j} - (160\text{mm})\mathbf{k}$$

$$\mathbf{T} = T \frac{\overrightarrow{DC}}{DC} = -\frac{6}{7}T\mathbf{i} + \frac{3}{7}T\mathbf{j} - \frac{2}{7}T\mathbf{k}$$

$$\sum \mathbf{f} = 0$$

$$A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} + B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k} + \mathbf{T} - (294\text{N})\mathbf{j} = 0$$

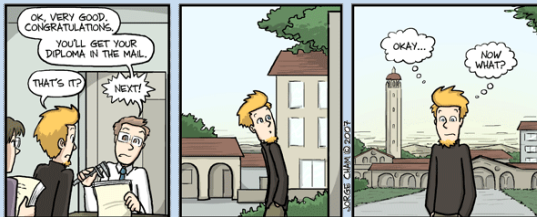
For the moment!



Some Hints for the Exam

Prepare yourself, All the sample problems and

- Section A: Equilibrium in two dimensions:
3/5, 3/10, 3/31 3/46, 3/55
- Section B: Equilibrium in three dimensions:
3/63, 3/73, 3/79, 3/81, 3/85
- Chapter Review: 3/98, 3/101, 3/107, 3/108, 3/109
- FBD, Equilibrium conditions, Question exams (in slides)



THIS MARKS THE END OF THE THIRD CHAPTER IN THE PHD SAGA...!