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Human and Robot Interaction Laboratory
Parallel Robots Courses
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Problem 1

Consider the 2-DOF parallel robots represented in Fig. 2.

1. Obtain the IKP;
2. Obtain the FKP;
3. Express the Jacobian matrix:
   - By differentiating the IKP
   - Using screw theory

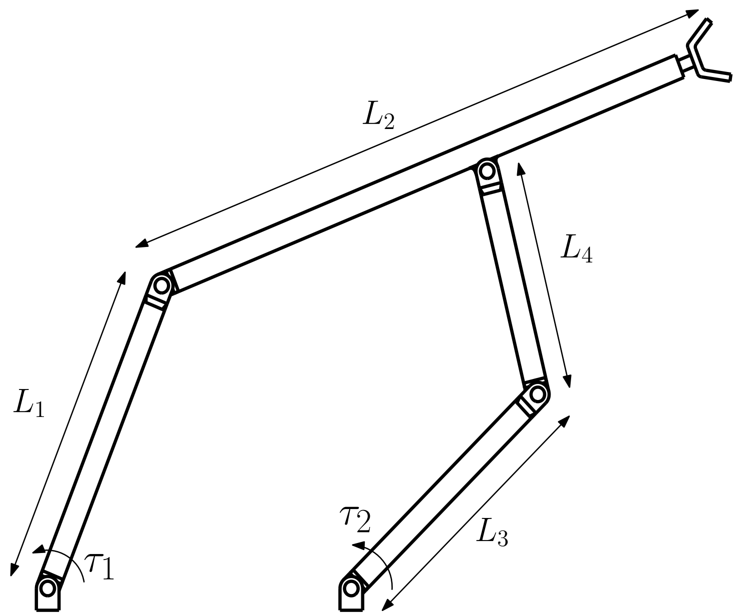


Figure 2.1: Planar manipulator with four revolute actuators, Problem 1.

	A_1	A_2	A_3	$\mathbf{e}_{\rho 1}$	$\mathbf{e}_{\rho 2}$	$\mathbf{e}_{\rho 3}$	C'_1	C'_2	C'_3
x	0	6	3	1	-0.5	-0.5	0	2	1
y	0	0	5.196	0	$\sqrt{3}/2$	$-\sqrt{3}/2$	0	0	1.732

Table 2.1: The design parameters of the 3-$\underline{\text{P}}$RR planar parallel robot, Problem 2.

4. Obtain the singularity expression;
5. Obtain the singularity configurations and justify them using screw theory;
6. Obtain the relation for the static problem;

Problem 2

Consider a 3-$\underline{\text{P}}$RR parallel robot, as depicted schematically in Fig. 2 with the design parameters as given in Table 2.1. Obtain the minimal-degree polynomial expression for the singularity loci for $\phi = 0$. To do so you should use the reasoning given in the course.

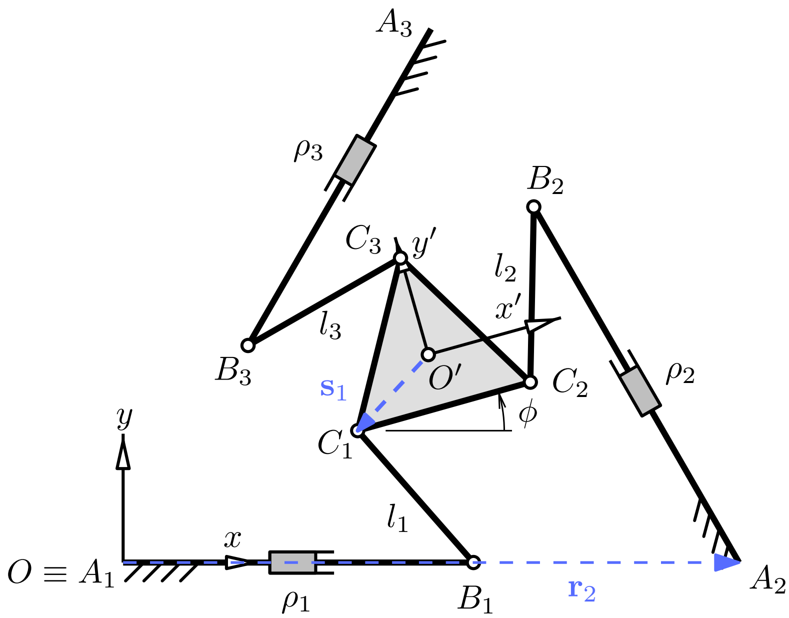


Figure 2.2: A 3-<u>P</u>RR planar robot, Problem 2.

Details About Problem 2

Assume that three reciprocal lines, called $\mathcal{L}_i$, intersect at point $\mathcal{M}(x_\mathcal{M}, y_\mathcal{M})$. Therefore, the vector, $\mathbf{b}_i$, $i = 1, 2, 3$, connecting a given point from $\mathcal{L}_i$, preferably a point where a joint is attached to, to $\mathcal{M}$ should be concurrent with $\mathcal{L}_i$. Therefore, the vector normal to $\mathcal{L}_i$, called $\mathbf{n}_{\mathcal{L}_i}$, must be also the normal vector of $\mathbf{b}_i$. If the aforementioned condition is satisfied, then it can be readily concluded that all $\mathcal{L}_i$, are concurrent and the mechanism exhibits a singularity. Following paragraphs substantiate the foregoing procedure in more details. The above geometric reasoning can be written mathematically as:

$$\mathbf{n}_{\mathcal{L}_i} \cdot \mathbf{b}_i = 0, \tag{2.1}$$

In the above, vector $\mathbf{n}_{\mathcal{L}_i}$ and $\mathbf{b}_i$ should be written with respect to the end-effector pose (position and orientation), (x, y, ϕ) upon substituting the inverse kinematic expression which leads to:

$$f_i = f(x, y, \phi, x_\mathcal{M}, y_\mathcal{M}, D_i), \quad i = 1, 2, 3 \tag{2.2}$$

Algorithm 1 The pseudo-code of the algorithm to obtain the minimal-degree polynomial representation of the singularity loci.

Input: D_i % design parameters of the planar parallel mechanism
Output: $f_{final}(x, y, \phi, D_i)$ % singularity loci
$\mathcal{L}_i$, $i = 1, 2, 3$ % reciprocal lines of each limb
$\mathbf{n}_{\mathcal{L}_i}$ % The vector normal to $\mathcal{L}_i$
$\mathbf{b}_i$ % the vector connecting a given point from $\mathcal{L}_i$ to point $\mathcal{M}$ (a preferably point in the space)
for i **from** 1 **to** 3 **do**
    $\mathbf{n}_{\mathcal{L}_i} \cdot \mathbf{b}_i = 0 \implies f_i = f_i(x, y, \phi, x_{\mathcal{M}}, y_{\mathcal{M}}, D_i)$
    $f_i' =$ `raise2square(isolate(`$f_i, \sqrt{\ }$`))` % first, isolate the square root and then raise to square both sides of equation
end for
f_{12}=`resultant`(f_1',f_2',$x_{\mathcal{M}}$)
f_{13}=`resultant`(f_1',f_3',$x_{\mathcal{M}}$)
f_{final}=`resultant`(f_{12},f_{13},$y_{\mathcal{M}}$)

where D_i is the set of design parameters of the i^{th} limb. From Eq. (2.2), three equations will be obtained which describes the singularity loci of the mechanism as a whole. Each of these equations includes just one square root, which usually appears upon substituting the inverse kinematic into eq. (2.1). One can simply isolate the square root and upon raising the whole expression to its square, the square root will be eliminated, leading to f_i', $i = 1, 2, 3$. Reaching this step, the three polynomials f_i', $i = 1, 2, 3$ are considered two by two and, at each stage, one variable is eliminated using *dyalitic elimination* concept, i.e., using the so-called *resultant* method. Thus coordinate of $\mathcal{M}_{(x_{\mathcal{M}}, y_{\mathcal{M}})}$, are subject to be eliminated to the end of obtaining a polynomial expression with respect to (x, y), called $\mathtt{f}_{final}$, known as the minimal-degree representation of singularity loci for a given orientation of the end-effector:

$$\begin{aligned}
f_{12} &= \mathtt{resultant}(f_1', f_2', x_{\mathcal{M}}), \\
f_{13} &= \mathtt{resultant}(f_1', f_3', x_{\mathcal{M}}), \\
f_{final} &= \mathtt{resultant}(f_{12}, f_{13}, y_{\mathcal{M}}).
\end{aligned}$$

In the above, `Resultant`(m, n, p) is a polynomial expression generated by eliminating the common variable p between the two polynomials m and n.

Finally, the procedure ends up with the expression of the singularity loci as, $f_{final} = f(x, y, \phi, D_i)$. In what follows, the algorithm is applied to 3-$\underline{P}$RR PM and results are presented.

The reasoning applied in the algorithm is: First f_1 and f_2 are considered together and $x_{\mathcal{M}}$ is vanished which leads to a new expression, named f_{12}. By the same token $x_{\mathcal{M}}$ will be vanished upon considering f_1 and f_3 which leads to f_{13}. Finally, f_{12} and f_{13} are considered together and $y_{\mathcal{M}}$ is eliminated through them. Now, the only remained variables in this expression are (x, y), i.e. the position of the end effector. This expression describes the mechanism singularity loci with respect to the end-effector variables.

Problem 3

For the following mechanisms obtain:

1. The constant-orientation workspace for $\phi = \{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\}$ using the the following approaches and compare your resluts:

    - Node search approaches (By combining the Newton-Raphson and bisection approaches);
    - The geometric constructive approach (Ask Amir Hossein for more details);
    - Using a CAD software;

2. The area of the constant-orientation workspace for $\phi = \{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\}$.

3. The evolution of the constant-orientation workspace for $\phi \in [0\ \frac{\pi}{2}]$

- The 3-$\underline{P}$RR planar parallel mechanisms as depicted in Fig. 2 with the following design parameters:
  $\lambda_a = 700$ mm, $\lambda_b = 1500$ mm, $\rho_{\min} = 200$ mm, $\rho_{\max} = 1500$ mm, $l = 300$ mm.

- The 3-$\underline{R}$RR planar parallel mechanisms as depicted in Fig. **??** with the folowing design parameters:
  $l_1 = 90$ mm, $l_2 = 90$ mm $r_b = 235$ mm, $r_e = 40$

Problem 4

Obtain the constant-orientation workspace and the corresponding volume of the Gough-Stewart platform where the geometric parameters and orientation of mobile platform are as given in Table 1 of the paper entitled "Determination of the Workspace of 6-DOF Parallel Manipulators". Click here to download the paper.

Note: In your report, you should present explicitly Figs. (4, 6–9) for the orientation of the end-effector given at the beginning of Section 5 page 334.

Use the following approaches and compare your results:

- Node search approaches (By combining the Newton-Raphson and bisection approaches);
- The geometric constructive approach (Ask Amir Hossein or Hossein Saadatzi for more details);
- Using a CAD software.

Problem 5

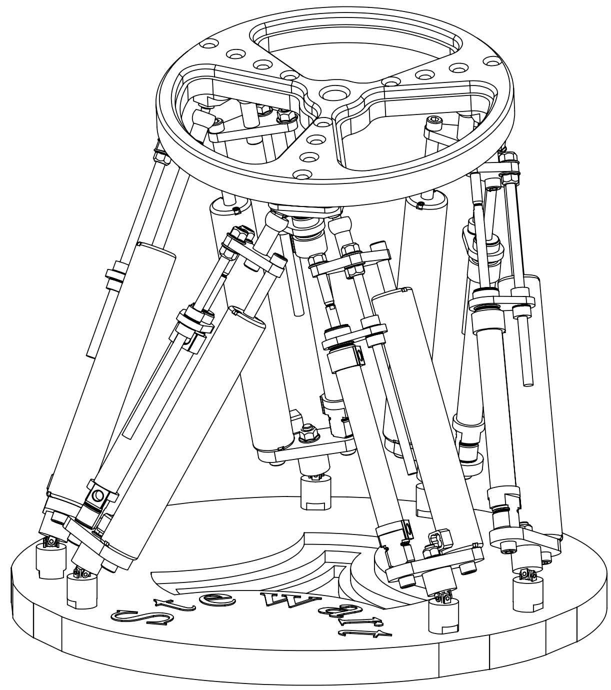
Solve the above problem for the Gough-Stewart platform, Fig. 2.3 built at TaarLab, University of Tehran. You should also consider limit joints and mechanical interferences. For dimensions discuss with Mojtaba Yazadani and Mahmoud Ghafouri.

Problem 6

Solve Problem 4 by considering Tripteron, Fig. 2.4, the one built at TaarLab, as case study. You should also consider limit joints and mechanical interferences. For dimensions discuss with Mojtaba Yazadani and Mahmoud Ghafouri.

Problem 7

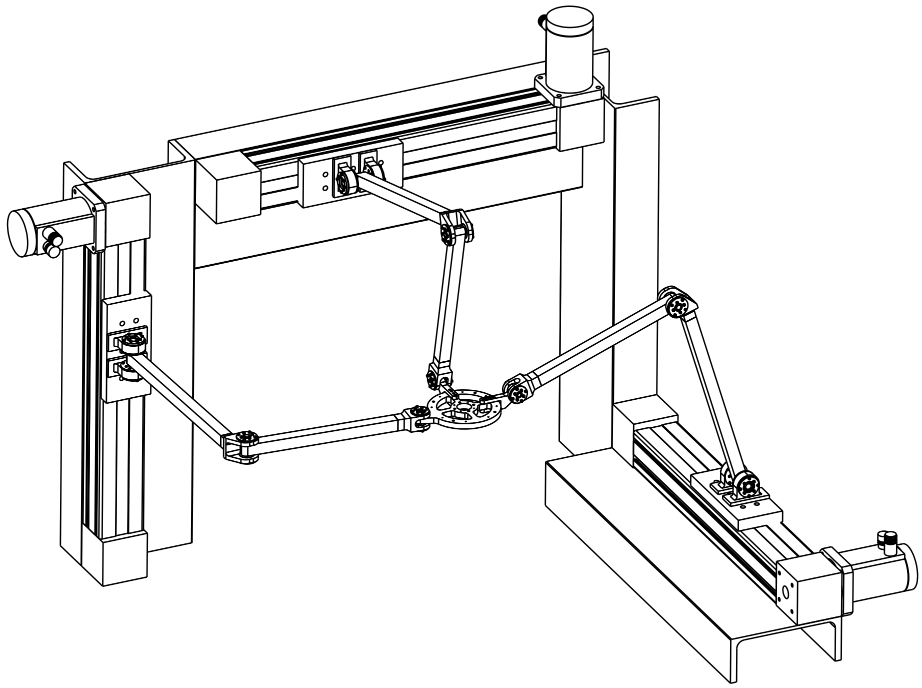
Using the Chebychev formula and screw theory find the DOF and motion pattern of the Agile 2-DOF. Solve the Static problem upon considering just the mass of the end-effecotr. For dimensions discuss with Esmail Rostami.



(a) CAD model

(b) Prototype

Figure 2.3: The Gough-Stewart Platform Built at TaarLab, University of Tehran.



(a) CAD model

(b) Prototype

Figure 2.4: The Tripteron Built at TaarLab, University of Tehran.

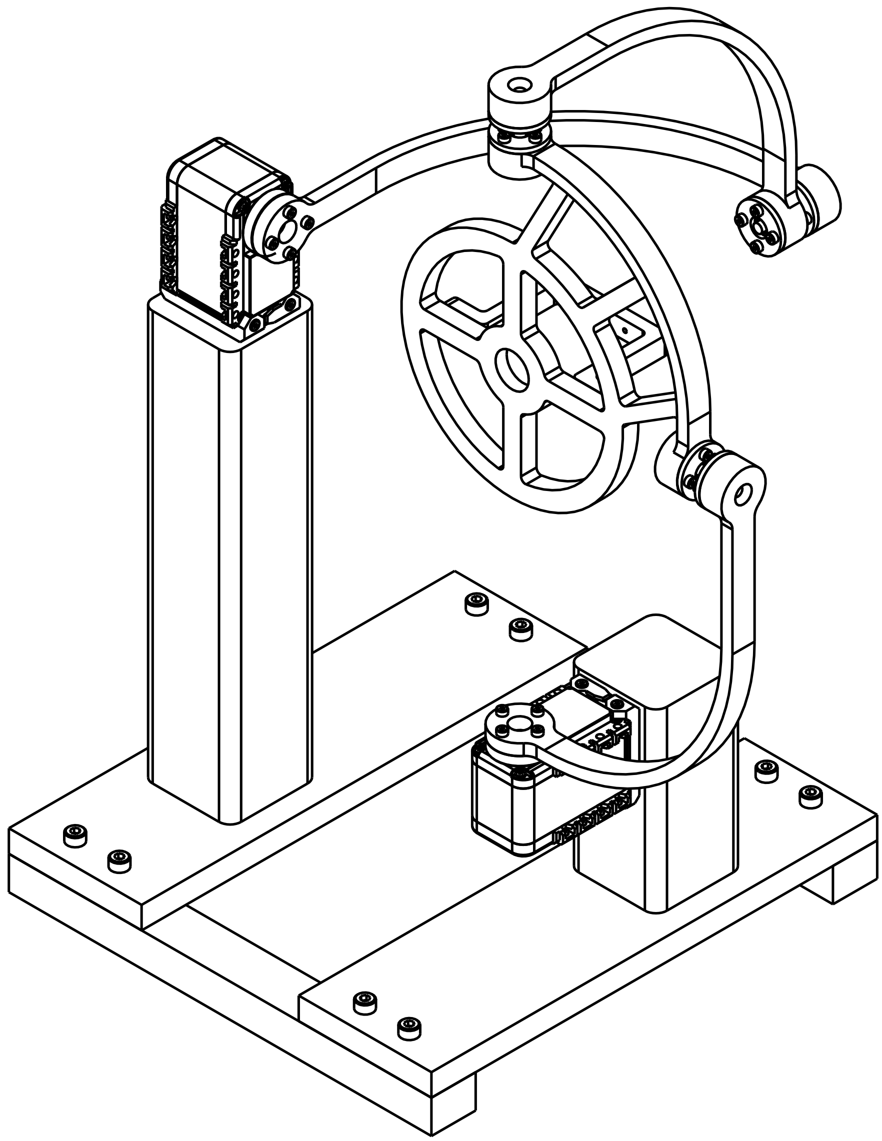


Figure 2.5: Agile Eye 2-DOF, Problem 7.