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Human and Robot Interaction Laboratory
Advanced Robotics
Mathematics Background

Problem 1

Given the following 3×3 matrix:

$$\mathbf{R} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix} \tag{1.1}$$

1. Show that it is a rotation matrix
2. Determine a unit vector that defines the axis of rotation and the angle (in degrees) of rotation.
3. What are the Euler parameters representing $\mathbf{R}$?

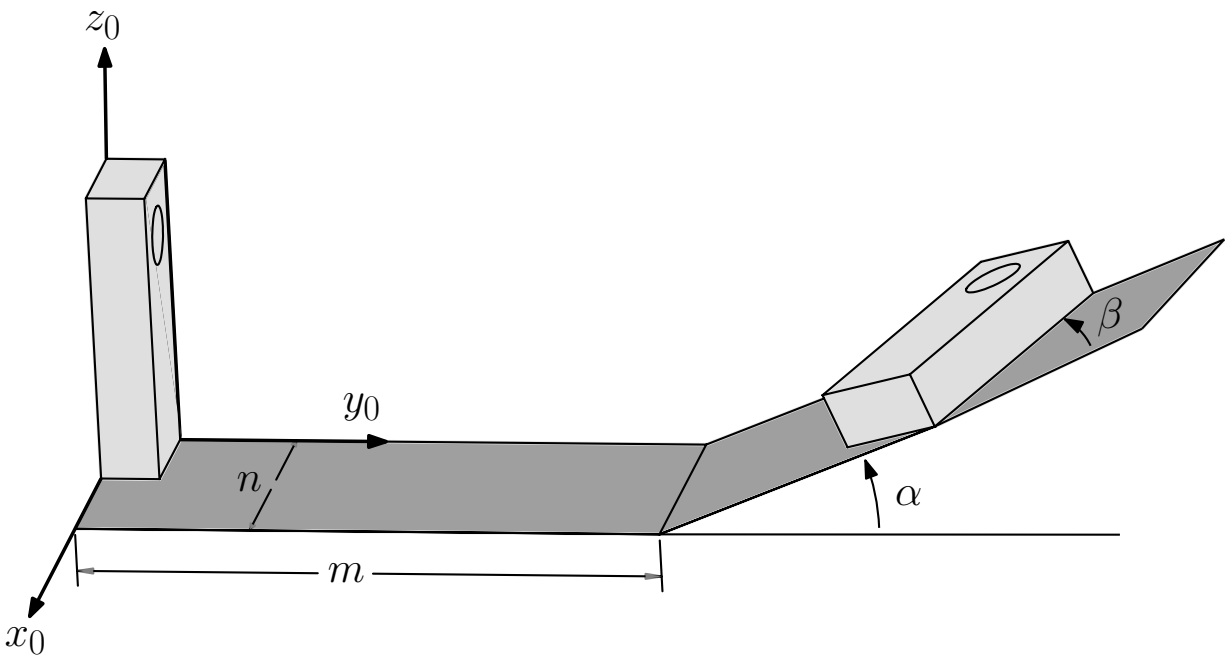

Figure 1.1: Schematic for Problem 3.

Problem 2

Solve Exercise 2.29

Problem 3

In Fig. **??**, a plate is moved from the horizontal base to an inclined surface by a manipulator. With respect to the **i**, **j** and **k** bases, determine:

1. The rotation matrix describing this operation;
2. The axes of rotation and the corresponding rotation angle about this axes.

Problem 4

Consider the case of two particles of mass $m_1$and m_2 each attached at the end of a mass less rod of length l_1 and l_2, respectively. Moreover, the second

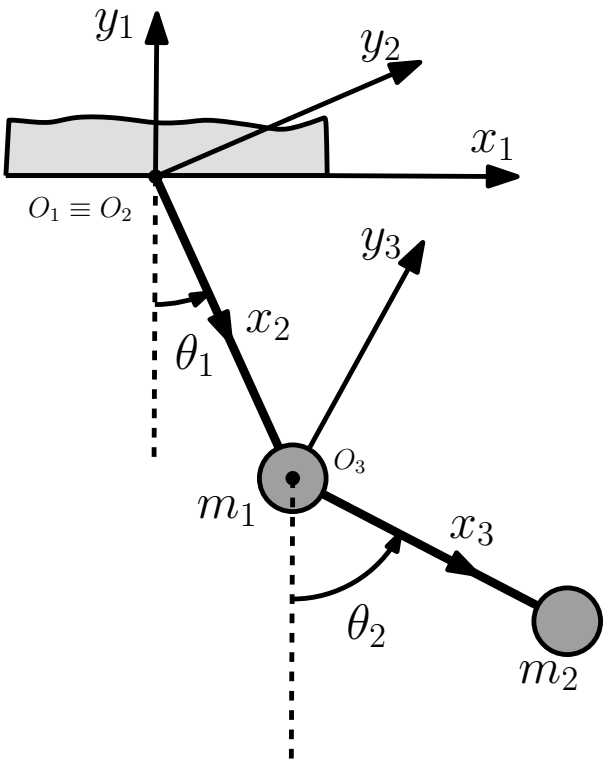

Figure 1.2: The double pendulum, Problem 4.

rod is also attached to the first particle. Derive the equations of motion for the two particles:

1. In frame $O_1(x_1, y_1)$
2. In frame $O_2(x_2, y_2)$
3. In frame $O_3(x_3, y_3)$

Assume, $m_1 = m_2 = 1$, $l_1 = l_2 = 2$ and $\theta_1 = \theta_2 = 0$. Compare your results with the one obtained from ADAMS.

Problem 5

Solve Exercise 2.35

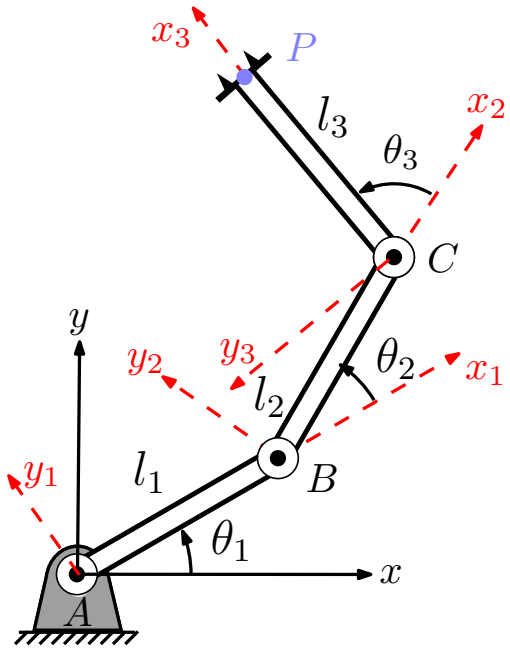

Figure 1.3: Schematic of a 3-DOF serial manipulator, Problem 6.

Problem 6

Consider the 3-DOF serial manipulator depicted schematically in Fig. **??**. The fixed based is denoted as O_{xy} where the mobile frames are represented as $O_{x_iy_i}$ fixed to the i^{th} limb. Find the absolute acceleration of point P:

- with respect to the fixed frames
- with respect to the frame 1
- with respect to frame 2
- with respect to frame 3

Problem 7

Solve Exercise 3.6

Problem 8

Solve Exercise 3.7

Problem 9

Solve Exercise 3.8